Section 2.0: Mathematical Programming

The next five chapters in the text focus on mathematical programming. The father of mathematical programming is George Dantzig. Between 1947 and 1949, Dantzig developed the basic concepts used for framing and solving linear programming problems. During WWII, he worked on developing various plans which the military called “programs.” After the war he was challenged to find an efficient way to develop and solve these programs.

Dantzig recognized that these programs could be formulated as a system of linear inequalities. Next, he introduced the concept of a goal. At that time, goals usually meant rules of thumb for carrying out a goal. For example, a navy admiral might have said, “Our goal is to win the war, and we can do that by building more battleships.” Dantzig was the first to express the selection of a plan to reach a goal as a mathematical function. Today it is called the *objective function*.

All of this work would not have had much practical value without a way to solve the problem. Dantzig found an efficient method called the simplex method. This mathematical technique finds the optimal solution to a set of linear inequalities that maximizes (profit) or minimizes (cost) an objective function.

Economists were excited by these developments. Several attended an early conference on linear programming and the simplex method called “Activity Analysis of Production and Allocation.” Some of them later won Nobel prizes in economics for their work. They were able to model fundamental economic principles using linear programming.

The first problem Dantzig solved was a minimum cost diet problem. The problem involved the solution of nine inequalities (nutrition requirements) with seventy-seven decision variables (sources of nutrition). The National Bureau of Standards supervised the solution process. It took the equivalent of one man working 120 days using a hand-operated desk calculator to solve the problem. Nowadays, a standard personal computer could solve this problem in less than one second. Excel spreadsheet software includes a standard “add-in” called “solver”, a tool for solving linear programming problems.

Mainframe computers became available in the 1950s and grew more and more powerful. This allowed many industries, such as the petroleum and chemical industries, to use the simplex method to solve practical problems. The field of linear programming grew very fast. This led to the development of non-linear programming, in which inequalities and/or the objective function are not linear functions. Another extension is called integer programming, in which the variables can only have integer values. Together, linear, non-linear and integer programming are called **mathematical programming**.

2.0.1 An Introductory Problem

In order to get a feel for mathematical programming, this chapter begins with a problem that has a concrete model. This model can be built from Lego pieces. When a mathematical model of a real world situation is constructed in symbolic form, it is often helpful to construct a physical or visual model at the same time. The role of the latter model is to help the model builder to understand the real-world situation as well as its mathematical model.

The Problem

A certain furniture company makes only two products: tables and chairs. The manufacturing of tables and chairs can be modeled using Lego pieces. To make a table requires two large and two small pieces, and a chair requires one large and two small pieces. Figure 2.0.1 shows a table and a chair made from Legos.

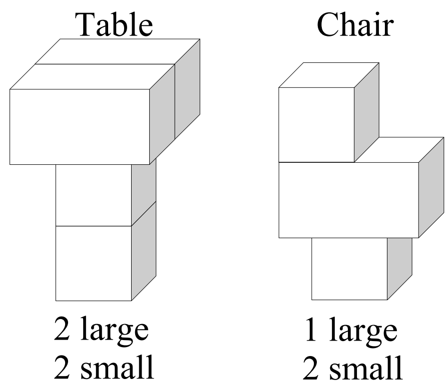


Figure 2.0.1: A Lego table and chair

If the resources needed to build tables and chairs were unlimited, the company would just manufacture as many of each as it thought it could sell. In the real world, however, resources are not unlimited. Suppose that the company can only obtain six large and eight small pieces per day. Figure 2.0.2 shows these limited resources.

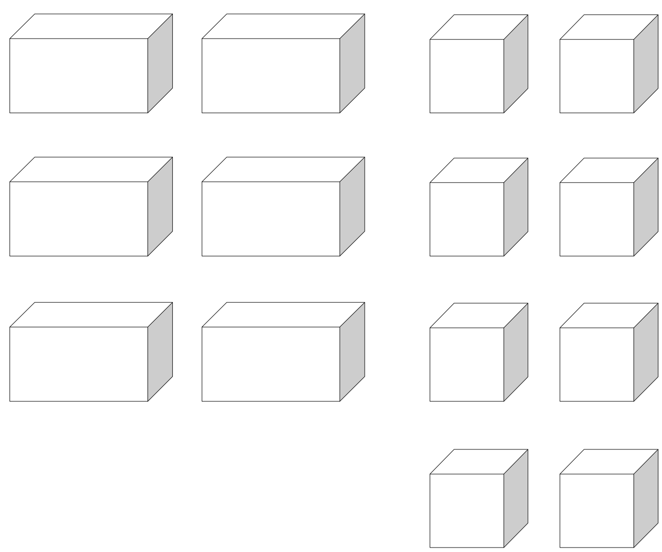


Figure 2.0.2: The furniture company's limited resources

The profit from each table is $16, and the profit from each chair is $10. The production manager wants to find the rate of production of tables and chairs per day that earns the most profit. **Production rate** refers to the number of tables and chairs this company can produce per day.

Q1. What do you think the production rates should be in order to generate the most profit?

Q2. Does the number of table and chairs produced each day have to be an integer value?

Q3. Using only eight small and six large Legos, build a physical model of this problem. If Legos are unavailable, draw pictures to explore some possibilities. Create several combinations of tables and chairs this company could make using your model.

Solving the Problem

There are many possible product mixes this company could make. A **product mix** is a combination of each product being manufactured. The various product mixes could be explored using the Lego model.

First, the company could begin by making as many tables as possible since the profit from a table is much greater than the profit from a chair. Each table requires two large pieces and two small pieces. There are only six large and eight small pieces available. Therefore, only three tables can be built. This generates 3($16) = $48 profit. There are two small pieces left over, but nothing can be built from them. Thus, $48 is the total profit if three tables (and no chairs) are built.

Three tables and zero chairs was one possible product mix. There could be other production rates that generate more profit.

No more than three tables could be made due to the limited resources available, and making three tables yielded a profit of $48. Now, suppose two tables are made. Manufacturing two tables uses four large and four small pieces. Now there are two large and four small pieces left over. These are just enough resources to build two chairs. The profit on two tables and two chairs would be 2($16) + 2($10) = $52. This is more profit than building three tables. However, the production manager wonders, "Is $52 the greatest profit possible? Is there another product mix that could generate more profit?"

Q4. In a Table 2.0.1, record other combinations of tables and chairs the company could produce. For each combination, write the production rate of tables, the production rate of chairs, and the profit for each possibility.

Production Rate of Tables	Production Rate of Chairs	Total Profit

Table 2.0.1: Exploring the total profit for each combination of tables and chairs

Q5. Which production rates generate the most profit?

Q6. Did any product mix yield a profit greater than $52?

It is impossible to find the total profit for every product mix because there are infinitely many possibilities. However, most likely no one in the class found a profit greater than $52. In the next section, you will learn how to know for certain you found the product mix with the greatest profit.

Notice that in Table 2.0.1 you used a set of similar equations to compute the profit for each possibility. These equations are the basis for the **objective function**. The two production rates *varied* across each possible product mix, and exploring these variations allows a *decision* about production to be made. Therefore, the production rates for tables and chairs are known as the **decision variables** for this problem.

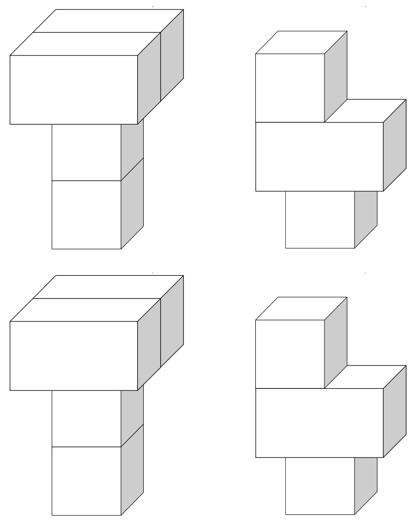


Figure 2.0.3: Two tables and two chairs yield the most profit

Because the profit has been optimized, the solution in Figure 2.0.3 is called the **optimal solution**. Besides the optimal solution, there are many other possible solutions. Although they are not optimal, each possible solution is still a **feasible** solution. Building four tables is an example of an **infeasible** solution.

Q7. Why is building four tables an example of an infeasible solution?

Q8. Give another example of an infeasible solution.

Stepping Beyond the Solution

Operations researchers understand that there is more to their work than merely finding solutions to problems. Once a solution is found, it must be interpreted. One sort of interpretation is called **sensitivity analysis**. Sensitivity analysis involves exploring how sensitive the solution is to changes in the parameters of the problem. For example, in the Lego problem above, one of the parameters of the problem is the availability of large pieces.

Q9. Would it make a difference if *seven* large pieces were available instead of six (there are still eight small pieces)? If so, what is the new optimal solution, and how much profit does it generate?

Q10. Would it make a difference if *nine* small pieces were available instead of eight (there are still six large pieces)? If so, what is the new optimal solution, and how much profit does it generate?

Q11. Would it make a difference if *seven* large pieces and *nine* small pieces were available? If so, what is the new optimal solution, and how much profit does it generate?

Growing the Problem

Suppose now that the furniture company has decided to dramatically expand production. Now it is able to obtain 27 small and 18 large Lego pieces per day. The profit on tables and chairs remains the same.

Q12. What should the daily production rates be in order to maximize profit?

Q13. Would it make a difference if 19 large pieces were available instead of 18 (there are still 27 small pieces)? If so, what is the new optimal solution, and how much profit does it generate?

Q14. Would it make a difference if 28 small pieces were available instead of 27 (there are still 18 large pieces)? If so, what is the new optimal solution, and how much profit does it generate?

Q15. Was this new problem easier or more difficult to solve than the original? Why?

Section 2.1: Computer Flips, a Junior Achievement Company

Junior Achievement (JA) is an educational program available worldwide. JA uses hands-on experiences to help young people understand the economics of life. In partnership with businesses and educators, JA brings the real world to students. The JA Company Program provides basic economic education for high school students by using support and guidance of volunteer consultants from the local business community. By organizing and operating an actual business, students learn how businesses function. They also learn about the structure of the free enterprise system and the benefits it provides.

Gates Williams is the production manager for Computer Flips, a Junior Achievement company. Computer Flips purchases a basic computer at wholesale prices and then adds a display, extra memory cards, extra USB ports, or a CD-ROM or DVD-ROM drive. The company also purchases these extra components at wholesale prices. The computers, with the added features, are then resold at retail prices.

Computer Flips produces two models: Simplex and Omniplex. The profit on each Simplex is $200, and on each Omniplex, the profit is $300. The Simplex model has fewer add-ons, so it requires only 60 minutes of installation time. The Omniplex has more add-ons and requires 120 minutes of installation time. Five JA students do all of the installation work. Each of them works 8 hours per week. Gates Williams must decide the rate of production per week of each computer model in order to maximize the company's weekly profit.

To make decisions such as the one Gates Williams faces, operations researchers use a technique known as **linear programming**. Answering the following questions will help you understand this technique.

2.1.1 Exploring the Problem

One way to approach the problem is to make some guesses and test the profit generated by each guess. For example, suppose Gates Williams decides the company should make 20 of each model.

Q1. How much profit would be generated?

Q2. Is there enough installation time available to make that number of each model?

Q3. Answer the same two questions if Gates Williams decides to make:
- a. 10 Simplex computers and 30 Omniplex computers
- b. 30 Simplex and 10 Omniplex

Q4. Can you find a product mix for which there is enough installation time?

Q5. How much profit do the production rates you found generate for the company?

2.1.2 Generalizing the Problem

Sometimes it is helpful to visualize things. The numbers, variables, and their relationships in a problem can be represented by a graph. Before graphing the Computer Flips problem, you must translate the information in the problem into mathematical statements—equations or inequalities.

First, let

x_1 represent the weekly production rate of Simplex computers and
x_2 represent the weekly production rate of Omniplex computers

The variables x_1 and x_2 are called decision variables because Gates Williams uses them to help make his decision. Mr. Williams's goal is to make as much money as possible. He does this by selling as many computers as he is able. Therefore, Mr. Williams can calculate his weekly profit (z) as a function of x_1 and x_2. Because the objective is to maximize profit, the profit function is called the objective function.

Q6. Write an equation for the profit (z) the company would earn in a week. [Hint: Look back at Section 2.1.1 and see how you calculated profit for 20 Simplex and 20 Omniplex computers.]

Q7. Write a mathematical statement in terms of x_1 and x_2 that describes the relationship between the installation time required to produce x_1 Simplex and x_2 Omniplex computers each week and the amount of available installation time each week. [Hint: Look back at Section 2.1.1 and see how you determined if there was enough installation time to produce 20 Simplex and 20 Omniplex computers.]

Q8. Can Computer Flips produce a negative number of either model?

Q9. Write two mathematical statements that describe your answer to the previous question.

The mathematical statements created in this section will be used to find the optimal solution in the following sections.

2.1.3 A Visual Approach

At this point, it should be clear that Gates Williams cannot decide to make any number of each model he chooses because there is only a certain amount of installation time available each week. That is, the available installation time *constrains* the number of Simplex and Omniplex computers that can be made each week. The inequality that captures this relationship (from Q7) is called a **constraint**. The other two inequalities (from Q9) express the fact that the decision variables in this problem cannot be negative. Thus, they are called **non-negativity constraints**.

These constraints can be graphed on a coordinate place. This graph gives a visual representation of the possible production rates for each computer model.

Q10. On the same coordinate axes, graph each of the three inequalities you wrote in the previous section (one from Q7 and one from Q9). For uniformity, place x_1 on the horizontal axis and x_2 on the vertical axis.

Q11. Give one point that satisfies all three inequalities.

Q12. Where are all of the points that satisfy all three inequalities?

Q13. What is the connection between the points identified in the previous question and the Simplex and Omniplex computers?

The points that satisfy each of the constraint inequalities represent a mix of Simplex and Omniplex computers that could be produced each week. Recall that this region of the coordinate system is called the feasible region, because those points represent feasible production mixes.

Q14. Choose any point in the feasible region, and compute the weekly profit that would be generated by producing that mix of Simplex and Omniplex computers.

Q15. Choose a second point in the feasible region that generates the same weekly profit as the first point.
a. Draw a line through the two points.
b. Write the equation for this line in terms of x_1 and x_2.

Every point on the line you have drawn generates the same weekly profit. For this reason, such a line is called a **line of constant profit**.

Q16. Suppose Computer Flips generates $6,000 of profit each week.
a. Write an equation to represent this situation.
b. Graph this equation.

Note that the points (0, 20), (15, 10), and (30, 0) are on the line you drew, and each coordinate pair generates a profit of $6,000 when substituted into the objective function.

Q17. For each of the following profits, write an equation and then graph that equation (on the same coordinate plane).
a. $0 profit
b. $3600 profit
c. $4800 profit
d. $7200 profit

Q18. What do you notice about the three lines you have drawn?

Q19. Which of the lines generates the largest weekly profit?

Q20. If you were to continue drawing lines in this way, where does the line that generates the largest weekly profit intersect the feasible region? What is the profit at that point?

2.1.4 Solving the Problem

Hopefully, the previous line of investigation has suggested that the point or points representing the largest possible weekly profit are close to the boundary of the feasible region. That is, in order to maximize profits, Computer Flips' production rates should be as large as possible, while still keeping within the available installation time.

Q21. Choose a point on the boundary of the feasible region, but not at a corner (vertex), and evaluate the profit there.

Q22. Continue to choose points on the boundary, but try to increase the amount of profit each time.

Q23. Finally, evaluate the profit at each of the corner points of the feasible region.

Q24. What is the relationship between the corner points and the feasible region?

Q25. At which of these points is the profit the greatest? How would you describe this point? Recall: The point at which the profit is maximized is called the optimal solution.

Notice that as the amount of constant profit increases, the lines are higher and further right in the first quadrant. Try to visualize a single line moving upward or to the right while its slope remains constant. The last point(s) in the feasible region that such a moving line touches will be optimal, because the profit is the greatest of any feasible points.

Q26. There is not always only one optimal solution.
Draw an example of a feasible region that could have more than one optimal solution.

2.1.5 Complicating the Problem

After several weeks of operation, one of the students in the sales department of Computer Flips does some market research. Based on this research, she decides that the company cannot sell more than 20 Simplex computers in any given week.

Q27. Write an inequality that expresses this market constraint.

Q28. Graph the new system of constraint inequalities.

Q29. What do you notice about the optimal solution you found earlier?

Q30. What is the optimal solution after adding the market constraint?

Now the students in the sales department of Computer Flips decide to extend the market research to the Omniplex model. On the basis of their research, they decide that Computer Flips cannot sell more than 16 Omniplex computers in any given week.

Q31. Write an inequality for this new market constraint.

Q32. Graph the new feasible region.

Q33. Does the previous optimal solution lie in the new feasible region?

Q34. What is the optimal solution after the addition of the second market constraint?

The students at Computer Flips notice that they are getting a lot of returns. Every computer that was returned had a problem with one of the add-ons. They realize that they need to test their finished products before shipping them. They decide to assign the task of testing the computers to only one of the student installers. To accommodate this change, the other four student installers agree to work 10 hours per week, so that the total available installation time remains 40 hours per week. The student who will do the testing also works 10 hours per week. It takes her 20 minutes to test a Simplex and 24 minutes to test an Omniplex.

Q35. Write an inequality for the testing constraint based on the information in the previous paragraph.

Q36. Graph the new feasible region.

Q37. Using the new feasible region, what is the optimal solution?

Q38. Why is it possible to have a non-integer solution?

2.1.6 Success Breeds—An Even More Complicated Problem

Computer Flips has some initial success, so the students are considering producing two additional models: Multiplex and Megaplex. Multiplex will have more add-ons than Simplex, but not as many as Omniplex. Each Multiplex will generate $250 profit. Megaplex, as the name implies, will have more add-ons than any of the other models. Each Megaplex will generate $400 profit.

Q39. What are the decision variables in the new problem? What do they represent?

Q40. Write an equation for the profit (z) the company would earn in a week.

The installation and testing times for each computer appear in Table 2.1.1. In addition, market research indicates that the *combined* sales of Simplex and Multiplex cannot exceed 20 computers per week, and the *combined* sales of Omniplex and Megaplex cannot exceed 16 computers per week.

	Simplex	Omniplex	Multiplex	Megaplex
Installation Time	60 min.	120 min.	90 min.	150 min.
Testing Time	20 min.	24 min.	24 min.	30 min.

Table 2.1.1: Installation and Testing times for all four computer models

Q41. Using the information above, formulate the constraints after the Multiplex and Megaplex models have been added to the product mix.

Q42. Is it possible to solve this problem by graphing? Why or why not?

In the next section, you will see another way to solve linear programming problems. In particular, the following section explores solving problems without graphing. You may wonder why this graphing approach cannot be used to solve every linear programming problem. If a problem contain three decision variables, it would be difficult for many people to visualize the graph. If a problem contains four or more decision variables, a graph is not even possible.

Section 2.2: SK8MAN, Inc.

SK8MAN, Inc. manufactures and sells skateboards. A skateboard is made of a deck, two trucks that hold the wheels (see Figure 2.2.1), four wheels, and a piece of grip tape. SK8MAN, Inc. manufactures the decks of skateboards in its own factory and purchases the rest of the components.

Figure 2.2.1: A skateboard truck

To produce a skateboard deck, the wood must be glued and pressed, then shaped. After a deck has been produced, the trucks and wheels are added to the deck to complete a skateboard. Skateboard decks are made of either North American maple or Chinese maple. A large piece of maple wood is peeled into very thin layers called veneers. A total of seven veneers are glued at a gluing machine and then placed in a hydraulic press for a period of time (see Figure 2.2.2). After the glued veneers are removed from the press, eight holes are drilled for the truck mounts. Then the new deck goes into a series of shaping, sanding, and painting processes. Figure 2.2.3 shows a deck during the shaping process.

Figure 2.2.2: Maple veneers in a hydraulic press

Figure 2.2.3: Shaping a deck

Currently, SK8MAN, Inc. manufactures two types of skateboards: Sporty (Figure 2.2.4, top) and Fancy (Figure 2.2.4, bottom).

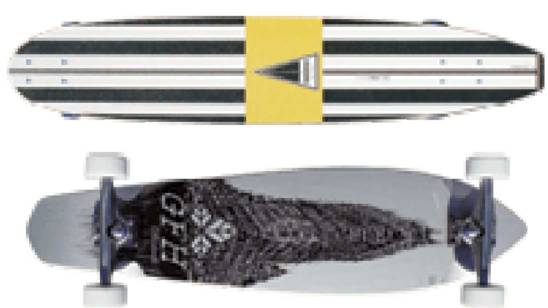


Figure 2.2.4: Sporty and Fancy skateboards

G. F. Hurley, the production manager at SK8MAN, Inc., needs to decide the production rate for each type of skateboard in order to make the most profit. Each Sporty board earns \$15 profit, and each Fancy board earns \$35 profit. However, Mr. Hurley might not be able to produce as many boards of either style as he would like, because some of the necessary resources, such as the North American maple and Chinese maple, are limited. That is, the production rates are *constrained* by the availability of the resources.

The Sporty board is a less expensive product, because its quality is not as good as the Fancy board. Chinese maple is used in the manufacture of Sporty decks. North American maple is used for Fancy decks. Because Chinese maple is soft, it is easier to shape. On average, it takes a worker 5 minutes to shape a Sporty board. However, a Fancy board requires 15 minutes to shape. G. F. Hurley needs to determine the production rates of Sporty and Fancy boards that will yield the maximum profit.

Q1. Develop a table to organize the information about Sporty and Fancy boards.

2.2.1 Problem Formulation

To find how to maximize profit, G. F. Hurley uses linear programming. The first step in the formulation of a linear programming problem is to define the decision variables in the problem. Let:

x_1 represent the weekly production rate of Sporty boards and
x_2 represent the weekly production rate of Fancy boards.

The decision variables are then used to define the objective function. This function captures the goal in the problem, which, in this case, is to *maximize* the company's profits per week. Therefore, the objective function should represent the weekly profit from the sale of the two different styles of skateboards. The variable z is used to represent the amount of profit SK8MAN, Inc. earns per week.

Now, since the profit for each style of skateboard is known (\$15 and \$35, respectively), G. F. Hurley writes the objective function by expressing the profit (z) in terms of the decision variables (x_1 and x_2):

Maximize: $z = 15x_1 + 35x_2$.

The last step in the formulation of the problem is to represent any constraints in terms of the decision variables. G. F. Hurley cannot just decide to make as many boards as he wants, because the number made is *constrained* by the available shaping time. Therefore, shaping time will be a constraint.

Suppose SK8MAN, Inc. is open for 8 hours a day, 5 days a week, which is a 40-hour workweek. However, since the information about shaping time is expressed in minutes, 40 hours is converted to 2,400 minutes. If SK8MAN, Inc. makes x_1 Sporty boards and x_2 Fancy boards per week, they use $5x_1 + 15x_2$ minutes of shaping time.

For example, making 100 Sporty boards and 150 Fancy boards would take 5(100) + 15(150) = 2,750 minutes. Note that since 2,750 minutes is greater than 2,400 minutes, this production mix is not feasible.

Thus, the shaping time constraint is:

$$5x_1 + 15x_2 \leq 2400$$

There are also two not-so-obvious but completely logical constraints. G. F. Hurley knows the production rate cannot be a negative number for either type of skateboard, so he writes the non-negativity constraints: $x_1 \geq 0$ and $x_2 \geq 0$.

The complete linear programming formulation looks like this:

Decision Variables

Let: x_1 = the weekly production rate of Sporty boards
x_2 = the weekly production rate of Fancy boards
z = the amount of profit SK8MAN, Inc. earns per week

Objective Function

Maximize: $z = 15x_1 + 35x_2$

Constraints

Subject to:

Shaping Time: $5x_1 + 15x_2 \leq 2400$

Non-Negativity: $x_1 \geq 0$ and $x_2 \geq 0$

This formulated linear programming problem can now be solved graphically. To do so, G. F. Hurley sets up a coordinate plane with x_1 as the horizontal axis and x_2 as the vertical axis. Then, he graphs the constraints, as shown in Figure 2.2.5.

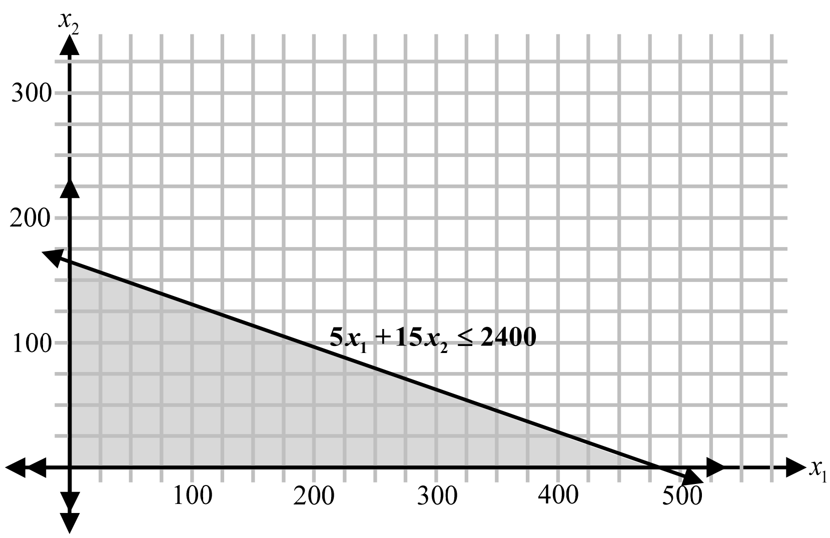


Figure 2.2.5: Graph of the system of constraints

G. F. Hurley recalls that every point in the shaded region satisfies all three constraints and is thus called the feasible region. Each ordered pair in the feasible region represents a combination of Fancy and Sporty boards that SK8MAN, Inc. could produce without violating any of the constraints. There are an infinite number of points in the feasible region, and the solution to the problem of maximizing profit is the one point that generates the most profit.

Rather than try to test an infinite number of points in the objective function, the optimal solution can be found by testing only a few points. This is due to the Corner Point Principle.

Corner Point Principle

If a linear programming problem has a unique optimal solution, it must occur at a corner point of the feasible region.

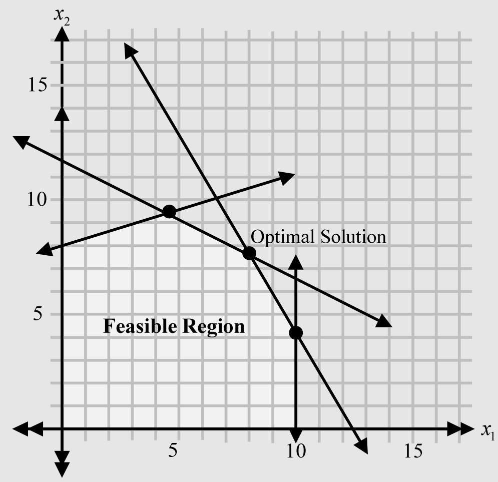


Figure 2.2.6: Example of an optimal solution at a corner point of the feasible region

The Corner Point Principle allows us to simply evaluate the objective function at each corner point of the feasible region. Instead of there being an infinite number of possibilities for the optimal solution, there are only as many possibilities as there are corners of the feasible region.

Therefore, G.F. Hurley tests only the corner points of the feasible region in the objective function, as shown in Table 2.2.1.

Point	Profit
(0, 0)	\$15(0) + \$35(0) = \$0
(480, 0)	\$15(480) + \$35(0) = \$7,200
(0, 160)	\$15(0) + \$35(160) = \$5,600

Table 2.2.1: Corner points and their profits

Based on this information, SK8MAN, Inc. should produce 480 Sporty boards and 0 Fancy boards each week. This product mix will generate a weekly profit of \$7,200.

2.2.2 Adding a New Constraint

G. F. Hurley just found out that the company that supplies the trucks for SK8MAN Inc.'s boards can provide at most 2,800 trucks per month. To make the problem easier, G. F. Hurley considers a month to be four weeks, and therefore there are 700 trucks available per week. Since each skateboard needs two trucks, this new information represents another constraint.

The new complete linear programming formulation is as follows:

Decision Variables

Let: x_1 = the weekly production rate of Sporty boards
x_2 = the weekly production rate of Fancy boards
z = the amount of profit SK8MAN, Inc. earns per week

Objective Function

Maximize: $z = 15x_1 + 35x_2$

Subject to:

Constraints

Shaping Time: $5x_1 + 15x_2 \leq 2400$
Trucks: $2x_1 + 2x_2 \leq 700$
Non-Negativity: $x_1 \geq 0$ and $x_2 \geq 0$

Again, G. F. Hurley graphs the constraints, as shown in Figure 2.2.6.

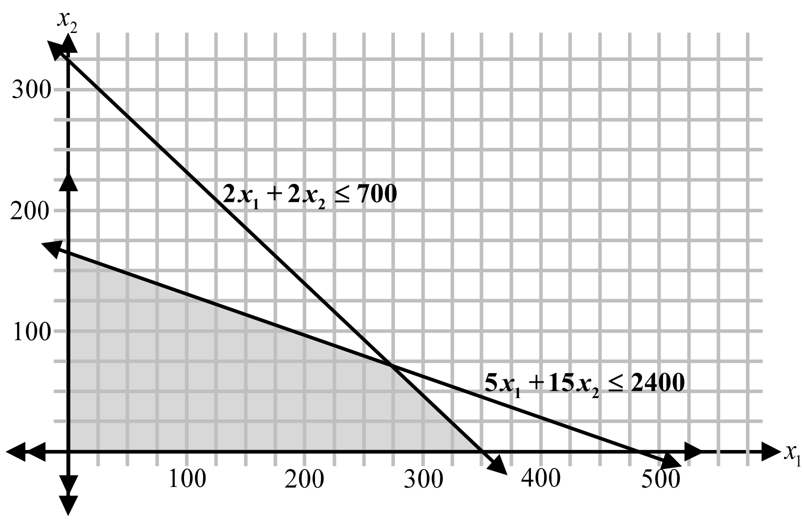


Figure 2.2.6: The feasible region after adding the truck constraint

The new constraint changes the feasible region. The previous optimal solution is no longer included. To find the new optimal solution, G. F. Hurley evaluates all the new corner points in the objective function, as seen in Table 2.2.2.

Point	Profit
(0, 0)	\$15(0) + \$35(0) = \$0
(350, 0)	\$15(350) + \$35(0) = \$5,250
(285, 65)	\$15(285) + \$35(65) = \$6,550
(0, 160)	\$15(0) + \$35(160) = \$5,600

Table 2.2.2: New corner points and their profits

Now G. F. Hurley can easily see that the maximum weekly profit SK8MAN, Inc. can earn is \$6,550, and the company does so by manufacturing 285 Sporty skateboards and 65 Fancy skateboards each week.

2.2.3 Adding a Third Constraint

The U.S. Congress recently enacted legislation regulating the consumption of North American maple by U.S. manufacturers. As a consequence, SK8MAN, Inc.'s supplier told the company that it can provide no more than 840 veneers per week. The law leads to a new constraint. Recall that to make a skateboard, seven veneers are glued together and then placed in a hydraulic press (see Figure 2.2.2). Also recall that North American maple is used only for Fancy decks (Sporty decks are made from Chinese maple).

G. F. Hurley develops the new complete linear programming formulation:

Decision Variables

Let: x_1 = the weekly production rate of Sporty boards
x_2 = the weekly production rate of Fancy boards
z = the amount of profit SK8MAN, Inc. earns per week

Objective Function

Maximize: $z = 15x_1 + 35x_2$

Subject to:

Constraints

Shaping Time:	$5x_1 + 15x_2 \le 2400$
Trucks:	$2x_1 + 2x_2 \le 700$
North American Maple:	$7x_2 \le 840$
Non-Negativity:	$x_1 \ge 0$ and $x_2 \ge 0$

Again, G. F. Hurley graphs the constraints, as shown in Figure 2.2.7.

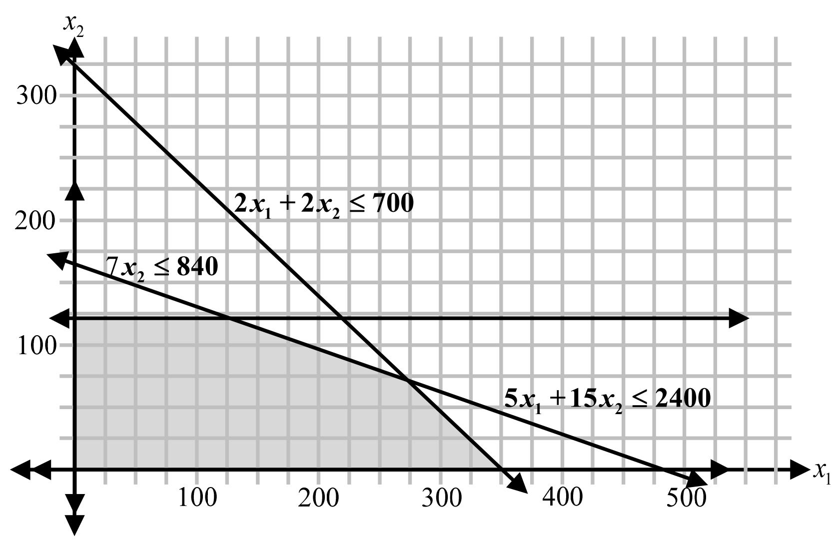


Figure 2.2.7: The feasible region after adding the North American maple constraint

When the new constraint is graphed, the feasible region changes, but the previous optimal solution (285 Sporty boards and 65 Fancy boards) is still included. Applying the corner point principle confirms that the maximum profit is unchanged because the optimal solution without the North American maple constraint remains in the feasible region after the North American maple constraint is added to the formulation. Table 2.2.3 shows the corner point calculations with the new constraint added to the formulation.

Point	Profit
(0, 0)	\$15(0) + \$35(0) = \$0
(350, 0)	\$15(350) + \$35(0) = \$5,250
(285, 65)	\$15(285) + \$35(65) = \$6,550
(120, 120)	\$15(120) + \$35(120) = \$6,000
(0, 120)	\$15(0) + \$35(160) = \$5,600

Table 2.2.3: Evaluating the objective function at each corner point of the new feasible region

Therefore, the optimal solution remains at 285 Sport boards and 65 Fancy boards. Since the North American maple constraint has no effect on the optimal product mix, it is called a **non-binding** constraint. The optimal product mix uses only 65(7) = 455 of the 840 available North American maple veneers (because the Sporty boards do not use North American maple, and the Fancy boards use 7 North American maple veneers per board). Not all of the available resource is expended in producing the optimal solution; thus there is a **slack** of 840 – 455 = 385. The ideas of non-binding constraints and slack will be explored throughout the chapter.

2.2.4 A Fourth Constraint

Finally, SK8MAN, Inc.'s Chinese maple supplier has decided to limit its exports and will deliver a maximum of 1,470 veneers per week. Now G. F. Hurley needs to determine the new mix of products that will maximize weekly profit. As before, this information leads to a new constraint, but the decision variables, objective function, and previous constraints remain the same. G. F. Hurley develops the new complete linear programming formulation:

Decision Variables

Let: x_1 = the weekly production rate of Sporty boards
x_2 = the weekly production rate of Fancy boards
z = the amount of profit SK8MAN, Inc. earns per week

Objective Function

Maximize: $z = 15x_1 + 35x_2$

Subject to:

Constraints

Shaping Time: $5x_1 + 15x_2 \leq 2400$
Trucks: $2x_1 + 2x_2 \leq 700$
North American Maple: $7x_2 \leq 840$
Chinese Maple: $7x_1 \leq 1470$
Non-Negativity: $x_1 \geq 0$ and $x_2 \geq 0$

Figure 2.2.8 shows the new graph of the constraints.

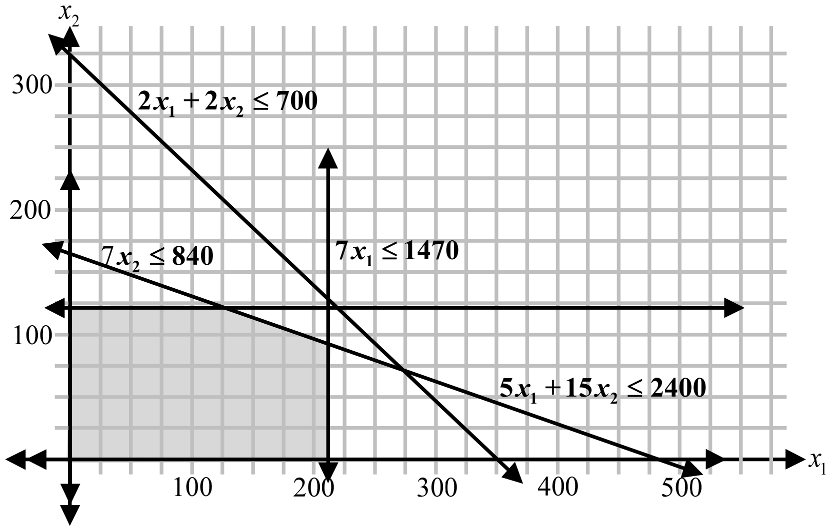


Figure 2.2.8: The feasible region after adding the Chinese maple constraint

As the graph in Figure 2.2.8 shows, the feasible region changes again. The previous optimal solution, (285, 65), is no longer feasible, so each corner point must be tested. Notice that the corner points created by the boundary of the Chinese maple constraint are new.

Point	Profit
(0, 0)	\$15(0) + \$35(0) = \$0
(210, 0)	\$15(210) + \$35(0) = \$3,150
(210, 90)	\$15(210) + \$35(90) = \$6,300
(120, 120)	\$15(120) + \$35(120) = \$6,000
(0, 120)	\$15(0) + \$35(160) = \$5,600

Table 2.2.3: Evaluating the objective function after the last constraint is added

Now SK8MAN, Inc.'s maximum profit is \$6,300 per week. The product mix that achieves that profit is 210 Sporty skateboards and 90 Fancy skateboards. Notice the tendency for maximum profit to decrease as the number of constraints increases.

2.2.5 Adding a Third Decision Variable

SK8MAN Inc. is introducing a new product—the Pool-Runner skateboard—which is made from Chinese maple. It is wider and shorter than the Sporty board so that it will be easy to use in a pool. It takes four minutes to shape a Pool-Runner board, and SK8MAN, Inc. earns \$20 for each one sold.

Q2. Develop a table to organize the information about Sporty, Fancy, and Pool-Runner boards.

G. F. Hurley needs to determine the new constraints and the optimal product mix. He begins by developing the new complete linear programming formulation:

Decision Variables

Let: x_1 = the weekly production rate of Sporty boards
x_2 = the weekly production rate of Fancy boards
x_3 = the weekly production rate of Pool-Runner boards
z = the amount of profit SK8MAN, Inc. earns per week

Objective Function

Maximize: $z = 15x_1 + 35x_2 + 20x_3$

Subject to:

Constraints

Shaping Time:	$5x_1 + 15x_2 + 4x_3 \leq 2400$
Trucks:	$2x_1 + 2x_2 + 2x_3 \leq 700$
North American Maple:	$7x_2 \leq 840$
Chinese Maple:	$7x_1 + 7x_3 \leq 1470$
Non-Negativity:	$x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$

Adding the third decision variable makes solving this problem graphically very difficult. Graphing the feasible region with three decision variables would require three dimensions. While it is possible to do so, visualizing such a feasible region is very difficult for most people. There are two other possible ways to solve linear programming problems involving three or more decision variables. The first way is to apply a paper-and-pencil technique called the Simplex Method. This method will not be described here. Instead, the use of a spreadsheet solver will be explored. A spreadsheet solver applies a computer procedure to solve linear programming problems. The following directions will walk you through the steps needed to use the Solver function in Microsoft Excel to solve this problem.

2.2.6 Using Excel Solver

The following steps are given in terms of Microsoft Office 2010. For information regarding earlier versions, see Appendix A.

Step 0: Add in Solver

Open Excel and go to “Data” menu. The Solver option should be at the top right of the menu (see Figure 2.2.9). If it is not available, it needs to be added. To add Solver, go to “Options” under the “File” menu (see Figure 2.2.10) and click on “Add-Ins” (see Figure 2.2.11). Next, choose the “Solver Add-in” and click “Go.” Finally, check “Solver Add-in” and click “OK” (See Figure 2.2.12).

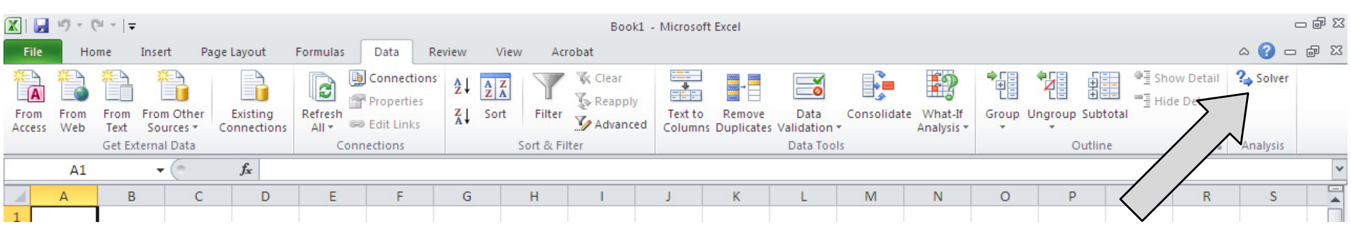


Figure 2.2.9: Location of Solver in Microsoft Excel 2010

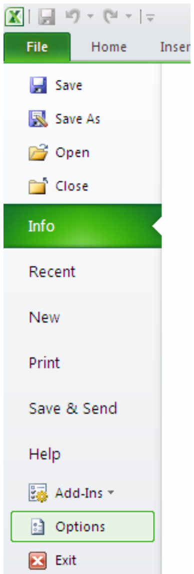


Figure 2.2.10: Choose "Options" under the "File" menu

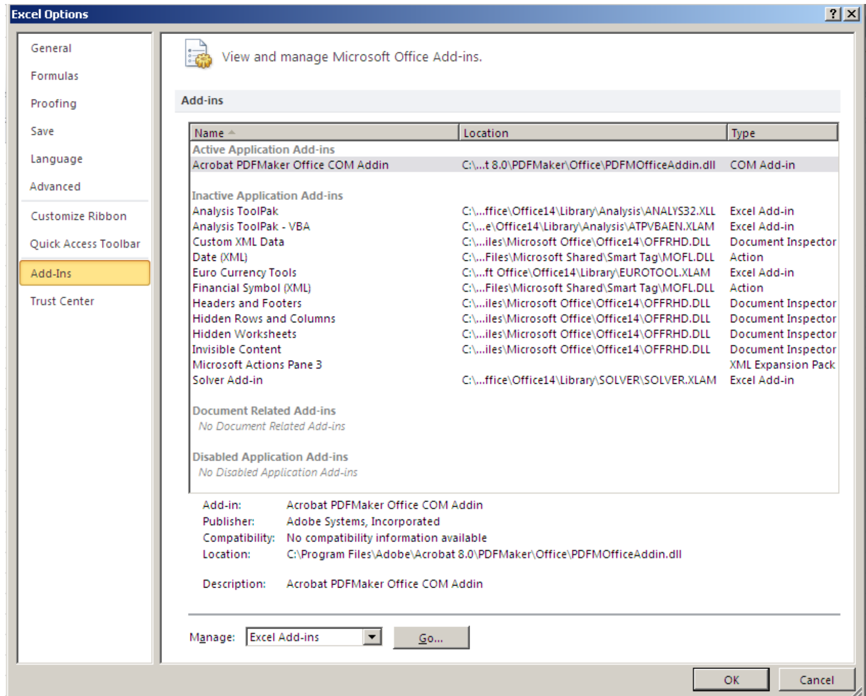


Figure 2.2.11: The Add-Ins menu in Microsoft Excel 2010

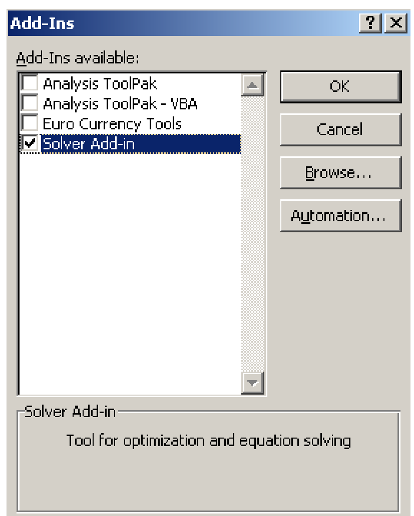


Figure 2.2.12: Choose "Solver Add-in" and click "OK"

Step 1: Set Up Spreadsheet
To set up the spreadsheet, keep in mind the complete linear programming formulation:

Decision Variables

Let: x_1 = the weekly production rate of Sporty boards
x_2 = the weekly production rate of Fancy boards
x_3 = the weekly production rate of Pool-Runner boards
z = the amount of profit SK8MAN, Inc. earns per week

Objective Function

Maximize: $z = 15x_1 + 35x_2 + 20x_3$

Subject to:

Constraints

Shaping Time:	$5x_1 + 15x_2 + 4x_3 \leq 2400$
Trucks:	$2x_1 + 2x_2 + 2x_3 \leq 700$
North American Maple:	$7x_2 \leq 840$
Chinese Maple:	$7x_1 + 7x_3 \leq 1470$
Non-Negativity:	$x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$

Begin by putting a title in column A, such as Chapter, Section, and type of problem (see Figure 2.2.13). This title will make things easier when coming back to this spreadsheet later.

Next, label cell A5 “Decision Variables” and cell A6 “Decision Values.” It is helpful to type in the description of the decision values as seen in Figure 2.2.13. The decision variables in this problem were the types of skateboards being produced at SK8MAN Inc. (Sporty, Fancy, and Pool-Runner). The empty cells for the decision values are treated as zero values. These cells are where Solver will put the values it computes for the decision variables once Solver is run. Because the values in these cells are continually changing, Solver calls them “changing cells.”

After the decision variables and values, the objective function will be written. It is helpful to write a short description of the objective function. In this example, G. F. Hurley wants to maximize profit, so the objective is to find profit.

Finally, the constraints are listed. The non-negativity constraints do not need to be written in the spreadsheet because Solver has an option that makes all decision values non-negative.

	A	B	C	D
1	Chapter 2: LP Maximization			
2	2.2 SK8MAN, Inc.			
3	Profit Maximization Problem			
4				
5	Decision Variable	**Sporty** (x1)	**Fancy** (x2)	**Pool-Runner** (x3)
6	Decision Values [weekly production rate]			
7				
8	Objective Function [Profit ($)]			
9				
10	**Constraints**			
11	Shaping Time (minutes)			
12	Truck Availability			
13	North American Maple Veneers			
14	Chinese Maple Veneers			

Figure 2.2.13: Setting up the problem formulation in an Excel spreadsheet

Step 2: Develop Formula for Objective Function
The objective function and each of the constraints need to be defined mathematically so that Solver will know what to compute.

First, recall that the objective function is $z = 15x_1 + 35x_2 + 20x_3$. That is, 15 is multiplied by the weekly production rate of Sporty boards, 35 is multiplied by the weekly production rate of Fancy boards, and 20 is multiplied by the weekly production rate of Pool-Runner boards.

Place the coefficients of the objective function in row 8 under the respective decision variable (see Figure 2.2.14).

The coefficients are multiplied by the cells containing the weekly production rate of each board, namely B6, C6, and D6, and then added together. This formula will be typed in cell E8. See Figure 2.2.14 for the formula for the objective function.

E8 f_x =B6*B8+C6*C8+D6*D8

	A	B	C	D	E
1	Chapter 2: LP Maximization				
2	2.2 SK8MAN, Inc.				
3	Profit Maximization Problem				
4					
5	Decision Variable	**Sporty** (x1)	**Fancy** (x2)	**Pool-Runner** (x3)	
6	Decision Values [weekly production rate]				
7					**Total Profit**
8	Objective Function [Profit ($)]	15	35	20	**0**
9					
10	**Constraints**				
11	Shaping Time (minutes)				
12	Truck Availability				
13	North American Maple Veneers				
14	Chinese Maple Veneers				

Figure 2.2.14: The formula for the objective function

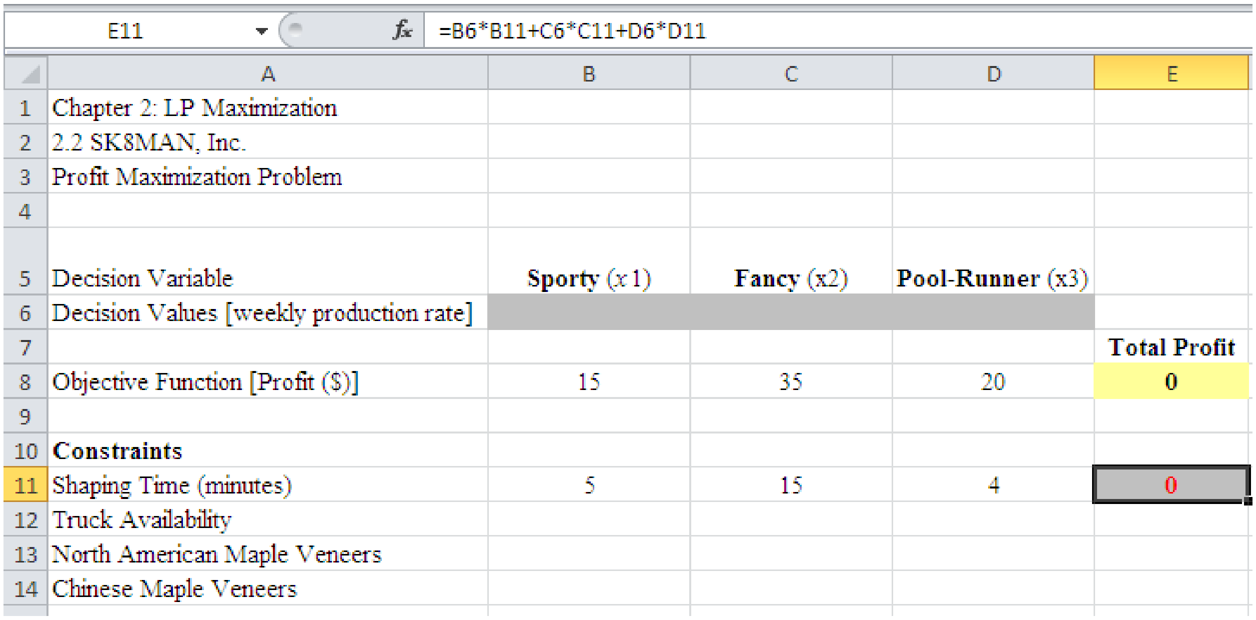
Q3. Compare the formula for the objective function in Excel with the objective function written algebraically (i.e., $z = 15x_1 + 35x_2 + 20x_3$).

Q4. Why did a zero appear in cell E8?

Step 3: Develop Formulas for Left-Hand Side of Constraints

The formulas for the constraints are written in the same way as the formula for the objective function. First consider the shaping time constraint: $5x_1 + 15x_2 + 4x_3 \leq 2400$. In this step, only consider the left hand side of the inequality.

Place the coefficients of the shaping time constraint inequality in row 10 under the respective decision variable (see Figure 2.2.15). Again, the coefficients are multiplied by the cells containing the weekly production rate of each board, namely B6, C6, and D6, and then added together. This formula will be typed in cell E10. Note that this expression computes the sum total of the shaping time that is consumed by a particular set of values of the three decision variables. See Figure 2.2.15 for the formula for the shaping time constraint.


E11 fx =B6*B11+C6*C11+D6*D11

	A	B	C	D	E
1	Chapter 2: LP Maximization				
2	2.2 SK8MAN, Inc.				
3	Profit Maximization Problem				
4					
5	Decision Variable	**Sporty** (x1)	**Fancy** (x2)	**Pool-Runner** (x3)	
6	Decision Values [weekly production rate]				
7					**Total Profit**
8	Objective Function [Profit ($)]	15	35	20	**0**
9					
10	**Constraints**				
11	Shaping Time (minutes)	5	15	4	0
12	Truck Availability				
13	North American Maple Veneers				
14	Chinese Maple Veneers				

Figure 2.2.15: The formula for the shaping time constraint

The formulas for the remaining constraints will look very similar to the shaping time constraint. Therefore, rather than type in each formula by hand, the *fill handle* will be used (see Chapter 1 for a review of the *fill handle*). Recall that in order to make some cells unchanging in a formula, dollar signs ($) must be used. Thus, the shaping time constraint should be revised to make the decision values unchanging. It should look as follows:

=$B$6 * B11 + $C$6 * C11 + $D$6 * D11

To obtain the left-hand side of each of the formulas for the remaining constraints, first type in the coefficients into the appropriate rows and then drag the fill handle into cells E12 through E14. The formulas for each of the remaining constraints look as follows:

Trucks: **=$B$6 * B12 + $C$6 * C12 + $D$6 * D12**
North American Maple: **=$B$6 * B13 + $C$6 * C13 + $D$6 * D13**
Chinese Maple: **=$B$6 * B14 + $C$6 * C14 + $D$6 * D14**

See Figure 2.2.16 for the remaining constraint formulas.

E11 =$B$6*B11+$C$6*C11+$D$6*D11

	A	B	C	D	E
1	Chapter 2: LP Maximization				
2	2.2 SK8MAN, Inc.				
3	Profit Maximization Problem				
4					
5	Decision Variable	**Sporty** (x1)	**Fancy** (x2)	**Pool-Runner** (x3)	
6	Decision Values [weekly production rate]				
7					**Total Profit**
8	Objective Function [Profit ($)]	15	35	20	**0**
9					
10	**Constraints**				
11	Shaping Time (minutes)	5	15	4	0
12	Truck Availability	2	2	2	0
13	North American Maple Veneers	0	7	0	0
14	Chinese Maple Veneers	7	0	7	0
15					

Figure 2.2.16: The spreadsheet with formulas for all four constraints added

Step 4: Type in Values for Right-Hand Side of Constraints

Next, the right-hand side of each constraint, which is the amount of each resource that is available, needs to be added. In addition, labels for the inequality sign should be added for convenience. This will help when setting up Solver.

For example, recall that the inequality for the shaping time constraint is: $5x_1 + 15x_2 + 4x_3 \leq 2400$. In the previous step, $5x_1 + 15x_2 + 4x_3$ (the left-hand side) was written in cell E11. This expression is less than or equal to 2400 because there are 2400 minutes available for shaping. Therefore, the symbol for "less than or equal to" should be placed in cell F11 (for ease, type "<=" rather than "≤"), and the value 2400 (the right-hand side) should be placed in cell G11.

Thus, the following should be added to cells F11 through F14 and cells G11 through G14 of the spreadsheet.

F11: <=	G11: **2400**
F12: <=	G12: **700**
F13: <=	G13: **840**
F14: <=	G14: **1470**

The spreadsheet should now look like the one in Figure 2.2.17. Notice that the amount of each resource that is consumed for a particular set of values of the decision variables will be displayed in column E, and those amounts must be less than or equal to the amount of each resource available that appears in column G.

	A	B	C	D	E	F	G
1	Chapter 2: LP Maximization						
2	2.2 SK8MAN, Inc.						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (*x*1)	**Fancy** (x2)	**Pool-Runner** (x3)			
6	Decision Values [weekly production rate]						
7					**Total Profit**		
8	Objective Function [Profit ($)]	15	35	20	0		
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	0	<=	2400
12	Truck Availability	2	2	2	0	<=	700
13	North American Maple Veneers	0	7	0	0	<=	840
14	Chinese Maple Veneers	7	0	7	0	<=	1470

Figure 2.2.17: The spreadsheet with the SK8MAN, Inc. problem completely formulated

Step 5: Open Solver

Before the problem can be solved, parameters need to be set up in the Solver program. These parameters include all of the parts of the problem formulation: the decision variables, the objective function, and the constraints. Solver needs to be told where in the spreadsheet each of these parameters is located.

First, click on the cell containing the objective function (cell E8).

Second, go to the Data menu and choose Solver (see Figure 2.2.9). A Solver Parameters window should come up with the target cell being $E$8 (see Figure 2.2.18). Notice that the target cell is the cell in which the objective function is defined. The dollar signs merely indicate that specific cell. When the Solver Parameters window is opened, if cell E8, containing the value of the objective function, is not already selected, select it now.

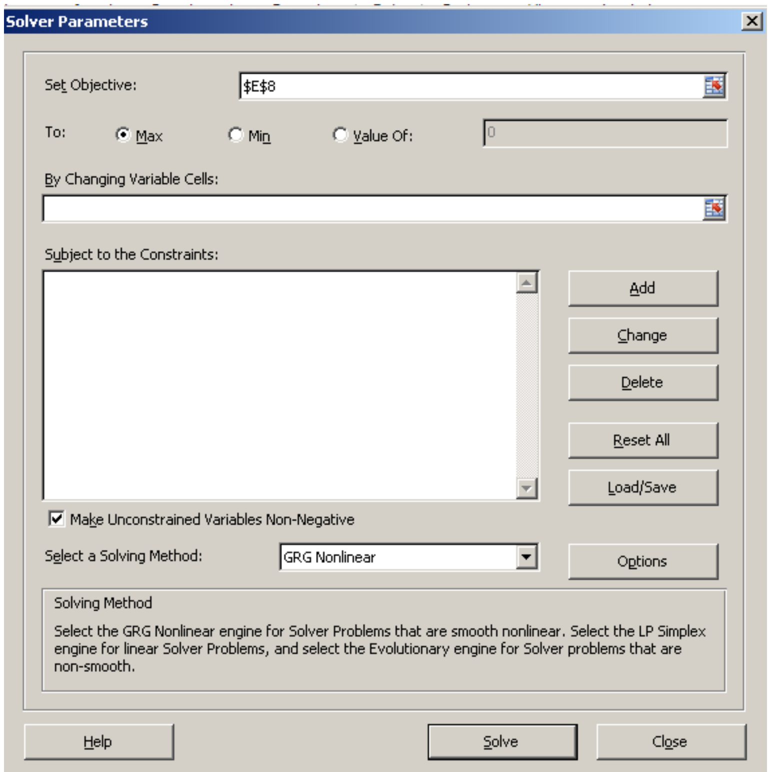


Figure 2.2.18: Beginning the Solver Parameters setup

Step 6: Choose Type of Linear Programing Problem
Recall that the objective for this problem is for SK8MAN, Inc. to *maximize* profit. Therefore, in the Solver Parameters window, make sure the “Max” circle is filled in.

Step 7: Choose Decision Variable Cells
Next, look at the “By Changing Variable Cells” title. Solver needs to be told that the decision variable cells are B6, C6, and D6. To do so, type in B6, C6, and D6 or use the shortcut B6:D6. Solver will add dollar signs in the cell names, and you can leave them as they are.

Step 8: Setting Up Constraints
Click inside the box labeled “Subject to the Constraints.” The constraints will be added, one at a time. Click “Add.” As shown in Figure 2.2.19, another window will appear titled “Add Constraint.”

The value of the formula in cell E11 should be less than or equal to the value in cell G11 (because the amount of shaping time used needs to be less than or equal to 2400 minutes). To do this, type E11 into “Cell Reference” and G11 into “Constraint.” The inequality symbol can be changed by using the drop-down menu. In this case it should be left as is (<=). Click “Add” and then continue to the next constraint.

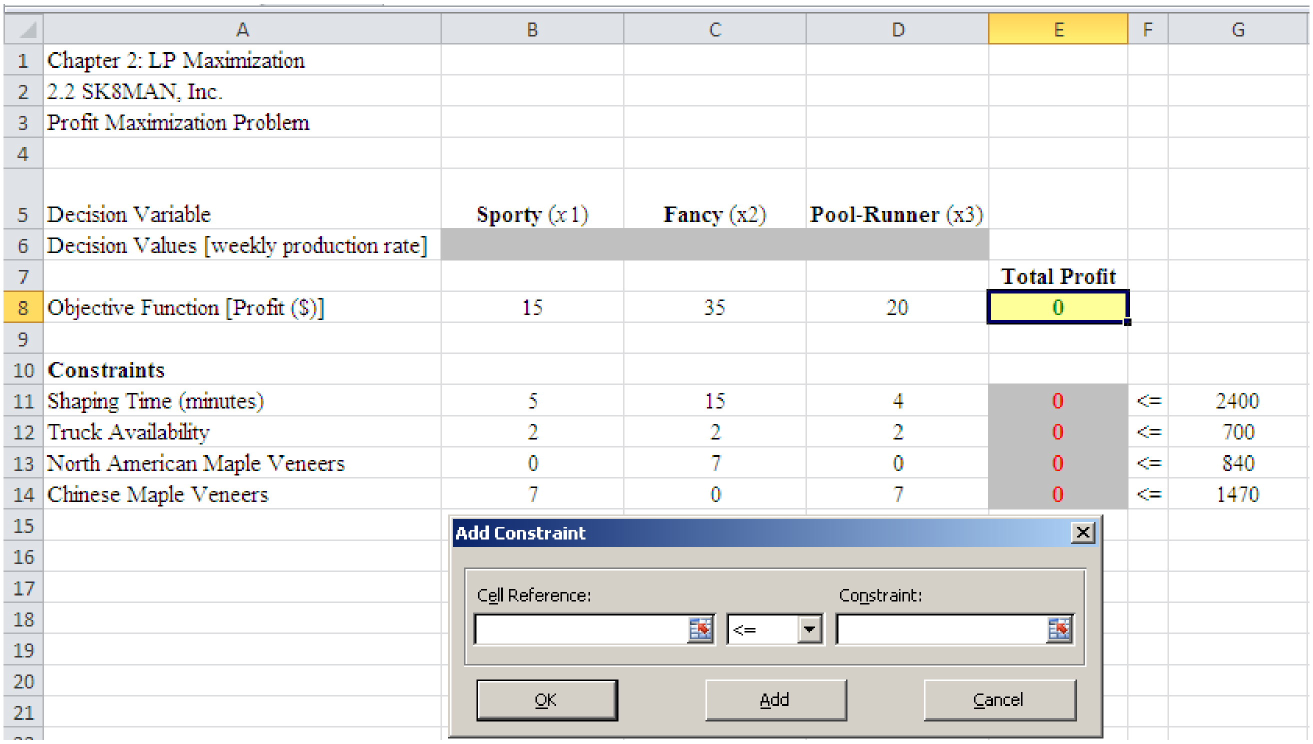


Figure 2.2.19: The "Add Constraint" window

For the Truck availability constraint, E12 should be less than or equal to G12, so type E12 into "Cell Reference" and G12 into "Constraint" and then click "Add." Continue in the fashion for the North American Maple Veneers constraint and the Chinese Maple Veneers constraint. After completing the Chinese Maple Veneers constraint, click "OK." When finished, the constraints should be listed in the "Subject to the Constraints" box in the Solver Parameters window (see Figure 2.2.20).

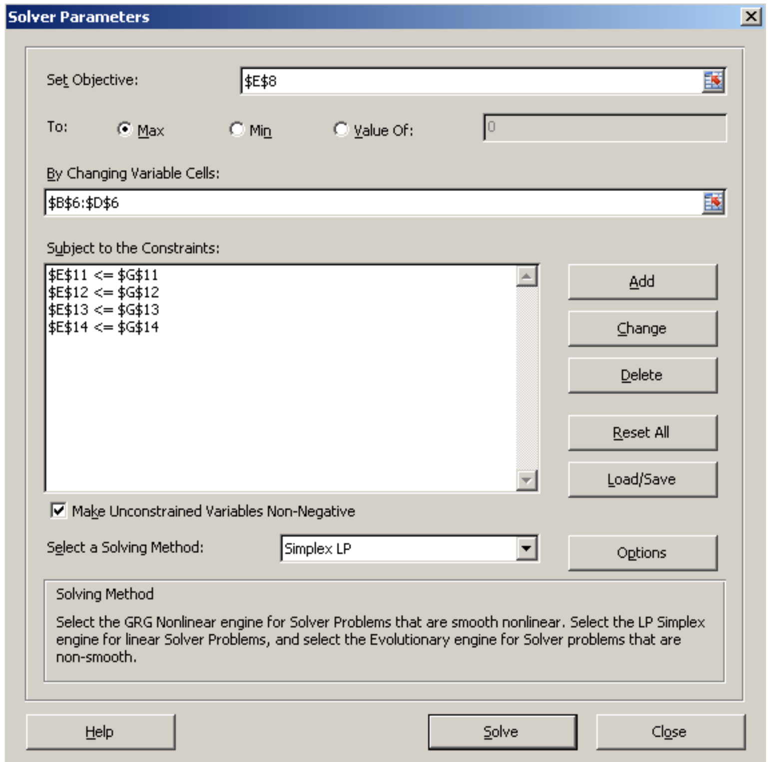


Figure 2.2.20: Continuing to set problem parameters in Solver

Step 9: Verify Non-Negativity Constraint
Notice that the non-negative constraints were not included because Solver has a shortcut for doing so. Under the "Subjects to the Constraints" box, make sure "Make Unconstrained Variables Non-Negative" is checked.

Step 10: Select Solving Method
To solve this linear programming problem, Solver must use the Simplex method. Therefore, in the "Select a Solving Method" drop-down menu, select "Simplex LP."

Step 11: Set Up Options Menu
Click on the "Options" button in the Solver Parameters window. In the "All Methods" tab, check the box that says "Use Automatic Scaling." Also check the box that says "Ignore Integer Constraints." Make sure the "Constraint Precision" is at most 0.000000001 (8 zeros after the decimal point). Note: if you find that your solution is not precise enough, you may need to make the constraint precision smaller. Finally, set the "Solving Limits." Set "Max Time (Seconds)" to 100 and "Iterations" to 100. Then click "OK." Figure 2.2.21 shows the appropriate settings for this linear programming problem.

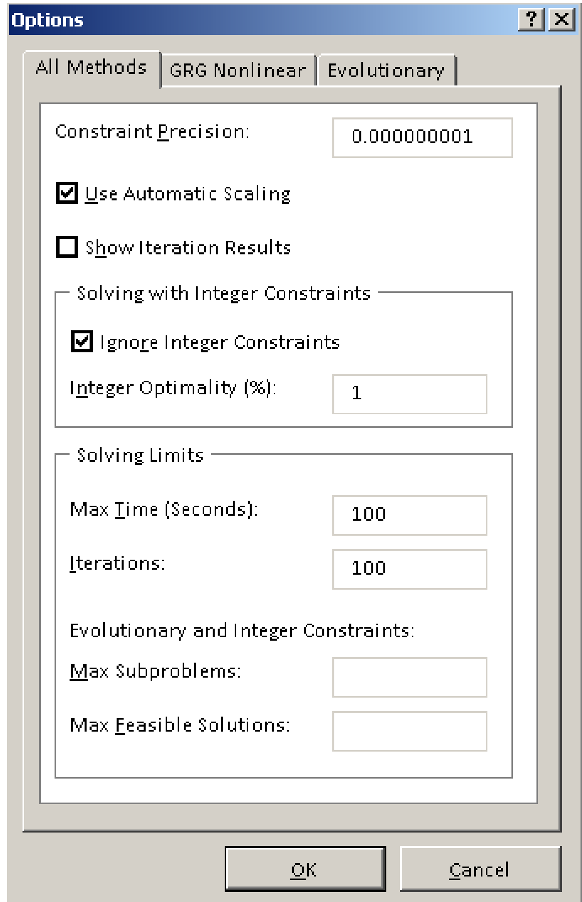


Figure 2.2.21: Adding the Solver Options

Step 12: Solve and Print Appropriate Reports
Finally, click "Solve." The Solver Results window appears, and the results can be seen in the spreadsheet, as shown in Figure 2.2.22.

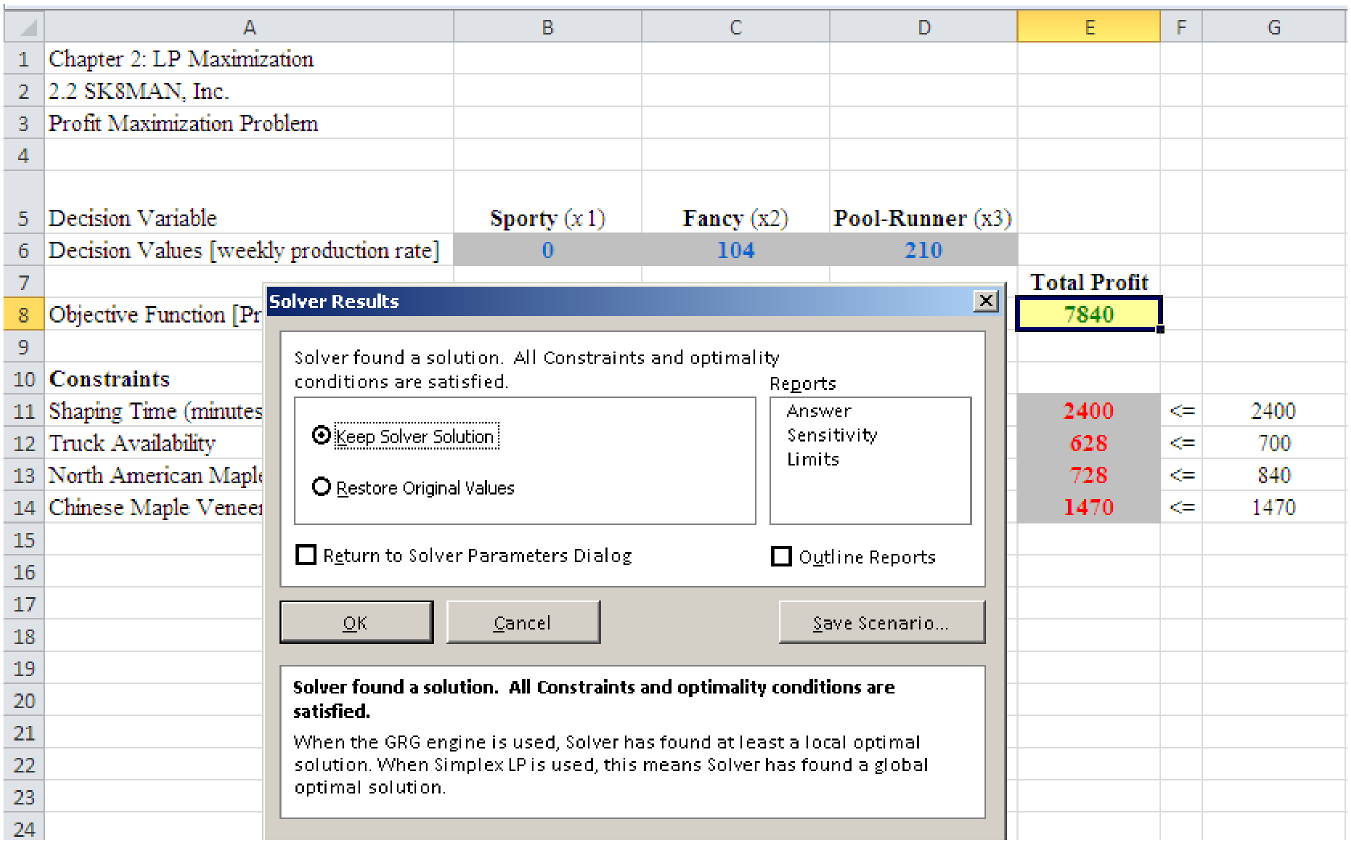


Figure 2.2.22: Solver has solved the three-decision-variable SK8MAN, Inc. problem

The Solver Results window shows various options to choose. For now, click "OK." Some of these options will be explored later.

Notice that x_1 has a final value of 0, x_2 has a final value of 104, and x_3 has a final value of 210. Thus, the optimal product mix calls for producing 0 Sporty skateboards, 104 Fancy skateboards, and 210 Pool-Runner skateboards per week. With this product mix, SK8MAN, Inc. will make a weekly profit of 15(0) + 35(104) + 20(210) = $7840.

Furthermore, notice that the workers at SK8MAN, Inc. will spend 5(0) + 15(104) + 4(210) = 2400 minutes shaping the three skateboards. Since there were only 2400 minutes available for shaping and they use all this time, this constraint is **binding**.

Q5. What other constraint is binding? How do you know?

Q6. What constraints are non-binding? How do you know?

Recall that the slack of each constraint can be found by subtracting the resources used from the available resources. For example, since 2400 minutes were available for shaping and 2400 minutes were used, the slack for the shaping time constraint is 2400 – 2400 = 0. Therefore, there is no time left over for shaping skateboards.

Q7. Calculate the slack for each of the remaining constraints. Interpret each value in terms of the context of the problem.

Q8. What is the relationship between slack and whether a constraint is binding?

To review, the steps for solving a maximization linear programming problem using Excel Solver are given in Table 2.2.4.

Step	Description
0	Add in Solver (skip this step once Solver has been added).
1	Set up the spreadsheet using the linear programming formulation.
2	Develop the formula for the objective function.
3	Develop the formulas for left-hand side of the constraints.
4	Type in the values for the right-hand side of the constraints.
5	Click on objective formula cell and choose Solver from the Data menu.
6	Verify that that "Max" circle is filled in.
7	Fill in the decision variable cells into the "By Changing Variable Cells" section.
8	Add constraints into the "Subject to the Constraints" section.
9	Verify that "Make Unconstrained Variables Non-Negative" is checked.
10	Choose "Simplex LP" from the "Select a Solving Method" drop-down menu.
11	Choose all appropriate options in the "All Methods" tab of the "Options" menu (see Figure 2.2.21).
12	Click "Solve." Interpret and analyze the results. Examine the Answer and Sensitivity Reports when desired.

Table 2.2.4: Steps for solving a maximization linear programming problem using Excel Solver

Q9. Based on these results, how many of each type of skateboard should SK8MAN, Inc. produce each week?

Q10. Under what conditions may G. F. Hurley decide to make a different product mix?

Q11. Summarize the main ideas of linear programming.

Section 2.3: The Pallas Sport Shoe Company

The Pallas Sport Shoe Company manufactures six different lines of sport shoes: High Rise, Max-Riser, Stuff It, Zoom, Sprint, and Rocket. Table 2.3.1 displays the amount of profit generated by each pair of shoes for each of these six lines. The production manager of the company would like to determine the daily production rates for each line of shoes that will maximize profit.

Product	High Rise	Max-Riser	Stuff It	Zoom	Sprint	Rocket
Profit	$18	$23	$22	$20	$18	$19

Table 2.3.1: Profit per pair for six lines of sport shoes

There are six main steps in the production of a pair of sport shoes at Pallas. Some of these steps can be seen in Figure 2.3.1.

1. **Stamping**: The parts that go together to form the upper portion of the shoe are cut using patterns on a large stamping machine. This process resembles cutting dough with a cookie cutter.
2. **Upper Finishing**: These parts are stitched or cemented together to form an upper, and holes for the laces are punched.
3. **Insole Stitching**: An insole is stitched to the sides of the upper.
4. **Molding**: The completed upper is then placed on a plastic mold, called a *last*, to form the final shape of the shoe.
5. **Sole-to-Upper Joining**: After the upper has been molded, it is cemented to the bottom sole using heat and pressure.
6. **Inspecting**: Finally, the shoe is inspected, and any excess cement is removed.

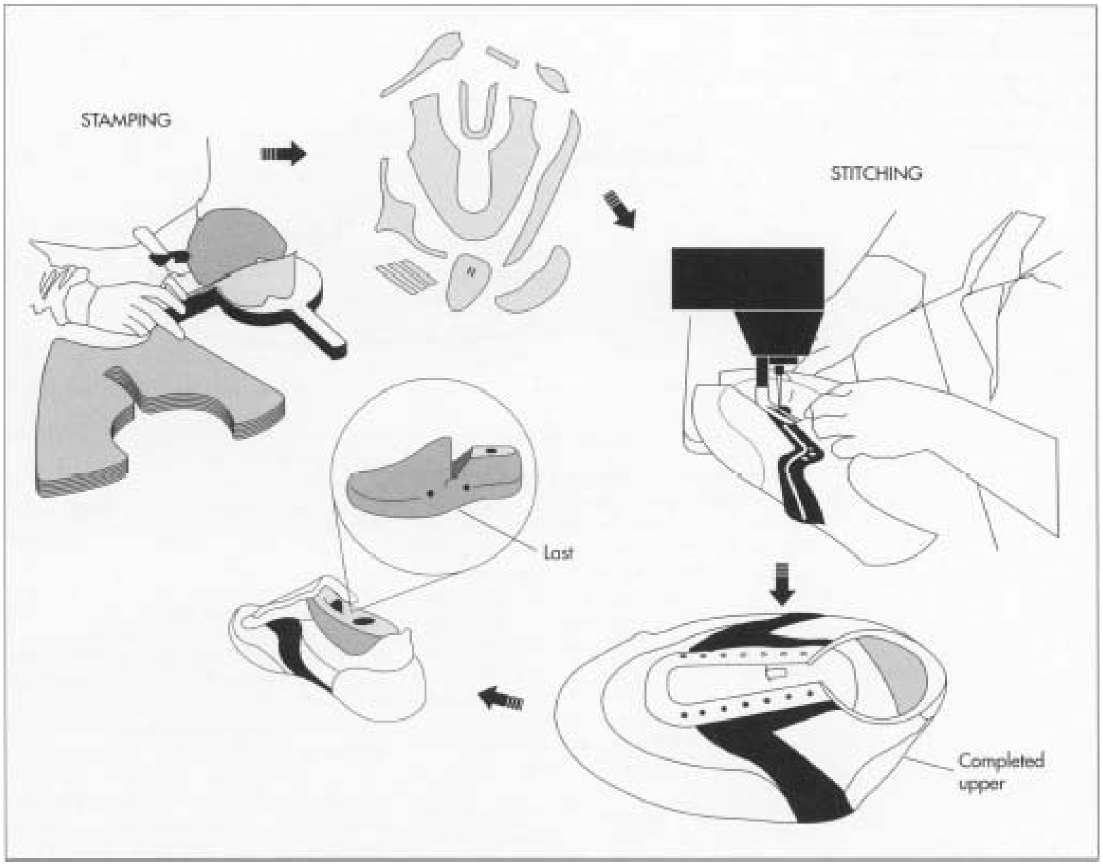


Figure 2.3.1: The steps in manufacturing a sport shoe

The time it takes to complete each of these steps differs across the six lines, and the total time available for each process constrains the daily production rates. Table 2.3.2 shows the time, in minutes, required for each of the six production steps for each of the six lines of sport shoe produced by Pallas as well as the total number of minutes available per day for each step.

	High Rise	Max-Riser	Stuff It	Zoom	Sprint	Rocket	Total Time Available
Stamping	1.25	2	1.5	1.75	1	1.25	420
Upper Finishing	3.5	3.75	5	3	4	4.25	1,260
Insole Stitching	2	3.25	2.75	2.25	3	2.5	840
Molding	5.5	6	7	6.5	8	5	2,100
Sole-to-Upper Joining	7.5	7.25	6	7	6.75	6.5	2,100
Inspecting	2	3	2	3	2	3	840

Table 2.3.2: Time, in minutes, per production step for each line of shoes and total time available

Q1. Based on the information in Tables 2.3.1 and 2.3.2, predict what the optimal solution will be for this problem. Explain your reasoning.

2.3.1 Problem Formulation

The linear programming formulation of the Pallas Sport Shoe Company problem appears below.

Decision Variables

Let:
x_1 = the daily production rate of High Rise
x_2 = the daily production rate of Max-Riser
x_3 = the daily production rate of Stuff It
x_4 = the daily production rate of Zoom
x_5 = the daily production rate of Sprint
x_6 = the daily production rate of Rocket
z = the amount of profit Pallas Sport Shoe Company earns per day

Objective Function

Maximize: $z = 18x_1 + 23x_2 + 22x_3 + 20x_4 + 18x_5 + 19x_6$

Subject to:

Constraints

Stamping Time: $1.25x_1 + 2x_2 + 1.5x_3 + 1.75x_4 + x_5 + 1.25x_6 \leq 420$
Upper Finishing Time: $3.5x_1 + 3.75x_2 + 5x_3 + 3x_4 + 4x_5 + 4.25x_6 \leq 1{,}260$
Insole Stitching Time: $2x_1 + 3.25x_2 + 2.75x_3 + 2.25x_4 + 3x_5 + 2.5x_6 \leq 840$
Molding Time: $5.5x_1 + 6x_2 + 7x_3 + 6.5x_4 + 8x_5 + 5x_6 \leq 2{,}100$
Sole-to-Upper Joining Time: $7.5x_1 + 7.25x_2 + 6x_3 + 7x_4 + 6.75x_5 + 6.5x_6 \leq 2{,}100$
Inspecting Time: $2x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6 \leq 840$
Non-Negativity: $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0$

Figure 2.3.2 contains this formulation in an Excel spreadsheet format for use with Solver.

	A	B	C	D	E	F	G	H	I	J
1	Chapter 2: LP Maximization									
2	2.3 Pallas Sport Show Company									
3	Profit Maximization									
4										
5	Decision Variable	**High Rise** (x_1)	**Max-Riser** (x_2)	**Stuff It** (x_3)	**Zoom** (x_4)	**Sprint** (x_5)	**Rocket** (x_6)			
6	Decision Values [daily production rate]									
7										**Total Profit**
8	Objective Function [Profit ($)]	18	23	22	20	18	19			**$0.00**
9										
10	**Constraints**							**Used**		**Available**
11	Cutting Time (minutes)	1.25	2	1.5	1.75	1	1.25	0	≤	420
12	Upper Finishing Time (minutes)	3.5	3.75	5	3	4	4.25	0	≤	1260
13	Insole Stitching Time (minutes)	2	3.25	2.75	2.25	3	2.5	0	≤	840
14	Molding Time (minutes)	5.5	6	7	6.5	8	5	0	≤	2100
15	Sole-to-Upper Joining Time (minutes)	7.5	7.25	6	7	6.75	6.5	0	≤	2100
16	Inspecting Time (minutes)	2	3	2	3	2	3	0	≤	840

Figure 2.3.2: An Excel spreadsheet formulation of the Pallas Shoe problem

2.3.2 Problem Solution

After solving this linear programming problem in Excel, an Answer Report can be generated, as shown in Figure 2.3.3. Figure 2.3.4 shows this Answer Report.

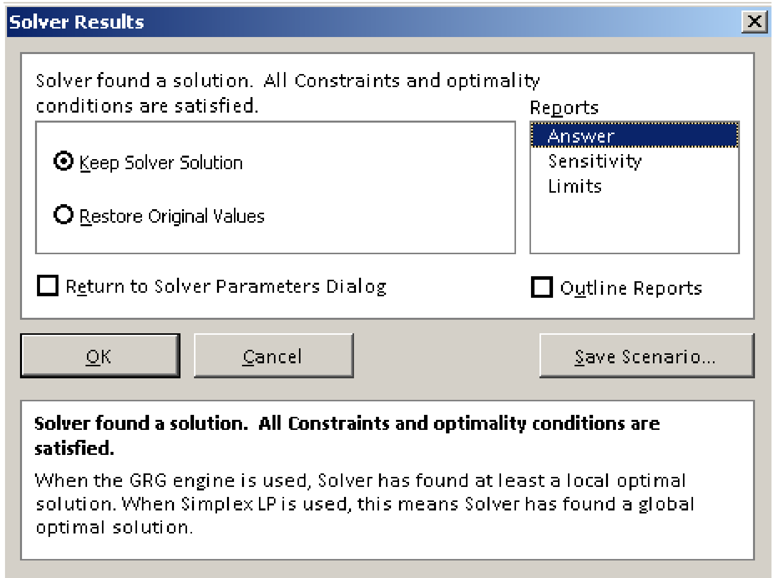


Figure 2.3.3: Generating an Answer Report in Excel Solver

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$J$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,132.57

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [daily production rate] High Rise (x1)	0	0	Contin
$C$6	Decision Values [daily production rate] Max-Riser (x2)	0	4.282944345	Contin
$D$6	Decision Values [daily production rate] Stuff It (x3)	0	45.12172352	Contin
$E$6	Decision Values [daily production rate] Zoom (x4)	0	72.32746858	Contin
$F$6	Decision Values [daily production rate] Sprint (x5)	0	104.9019749	Contin
$G$6	Decision Values [daily production rate] Rocket (x6)	0	89.82118492	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$11	Cutting Time (minutes) Used	420	$H$11<=$J$11	Binding	0
$H$12	Upper Finishing Time (minutes) Used	1260	$H$12<=$J$12	Binding	0
$H$13	Insole Stitching Time (minutes) Used	840	$H$13<=$J$13	Binding	0
$H$14	Molding Time (minutes) Used	2100	$H$14<=$J$14	Binding	0
$H$15	Sole-to-Upper Joining Time (minutes) Used	2100	$H$15<=$J$15	Binding	0
$H$16	Inspecting Time (minutes) Used	799.3421903	$H$16<=$J$16	Not Binding	40.65780969

Figure 2.3.4: Answer Report for Pallas Sport Shoe Company

Notice that the Answer Report is split into three sections: (1) Objective Cell (Max), (2) Variable Cells, and (3) Constraints.

Q2. Use the "Objective Cell (Max)" section of the Answer Report in Figure 2.3.4 to complete the following.
- a. Why is "$J$8" listed under "Cell?"
- b. Why do you think "Objective Function [Profit ($)] Total Profit" is listed under "Name?"
- c. Why do you think the "Original Value" is 0?
- d. What is the "Final Value" referring to?

Q3. Use the "Variable Cells" section of the Answer Report in Figure 2.3.4 to complete the following.
- a. What are the "Original Value" and "Final Value" columns referring to?
- b. Interpret the information given in the "Final Value" column in terms of the problem context.
- c. How do you think the production manager should handle the decimal values that appear in the "Final Value" column?

Q4. Use the "Constraints" section of the Answer Report in Figure 2.3.4 to complete the following.
- a. Interpret the information given in the "Cell Value" column in terms of the problem context.
- b. The first five constraints are binding and the sixth one is not. What does this mean in terms of the problem context?
- c. How are the values in the "Slack" column calculated?
- d. If you were only given the slack value for a constraint, how could you determine whether that constraint is binding?

Q5. Which of the six sport shoe lines should be produced, and at what daily rates, in order to maximize profit? (Approximate to two decimal places.)

Suppose Pallas Sport Shoe Company considers adding another five minutes of cutting time each day. Therefore, the cutting time constraint is changed from 420 to 425. The Answer Report for this new scenario is shown in Figure 2.3.5.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$J$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,160.47

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [daily production rate] High Rise (x1)	0	0	Contin
$C$6	Decision Values [daily production rate] Max-Riser (x2)	0	6.700179533	Contin
$D$6	Decision Values [daily production rate] Stuff It (x3)	0	48.89766607	Contin
$E$6	Decision Values [daily production rate] Zoom (x4)	0	74.55655296	Contin
$F$6	Decision Values [daily production rate] Sprint (x5)	0	100.4452424	Contin
$G$6	Decision Values [daily production rate] Rocket (x6)	0	85.86714542	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$11	Cutting Time (minutes) Used	425	$H$11<=$J$11	Binding	0
$H$12	Upper Finishing Time (minutes) Used	1260	$H$12<=$J$12	Binding	0
$H$13	Insole Stitching Time (minutes) Used	840	$H$13<=$J$13	Binding	0
$H$14	Molding Time (minutes) Used	2100	$H$14<=$J$14	Binding	0
$H$15	Sole-to-Upper Joining Time (minutes) Used	2100	$H$15<=$J$15	Binding	0
$H$16	Inspecting Time (minutes) Used	800.0574506	$H$16<=$J$16	Not Binding	39.94254937

Figure 2.3.5: Answer Report for Pallas Sport Shoe Company with 425-minute Cutting Time constraint

Q6. How does the Answer Report in Figure 2.3.5 differ from the one in Figure 2.3.4?

Q7. Is the first constraint still binding? Do you think Pallas Sport Shoe Company should add this extra five minutes of cutting time each day? Explain your reasoning.

Next, suppose Pallas Sport Shoe Company considers subtracting (rather than adding) five minutes of cutting time each day. Therefore, the cutting time constraint is changed from 420 to 415. The Answer Report for this new scenario is shown in Figure 2.3.6.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$J$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,104.67

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [daily production rate] High Rise (x1)	0	0	Contin
$C$6	Decision Values [daily production rate] Max-Riser (x2)	0	1.865709156	Contin
$D$6	Decision Values [daily production rate] Stuff It (x3)	0	41.34578097	Contin
$E$6	Decision Values [daily production rate] Zoom (x4)	0	70.0983842	Contin
$F$6	Decision Values [daily production rate] Sprint (x5)	0	109.3587074	Contin
$G$6	Decision Values [daily production rate] Rocket (x6)	0	93.77522442	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$11	Cutting Time (minutes) Used	415	$H$11<=$J$11	Binding	0
$H$12	Upper Finishing Time (minutes) Used	1260	$H$12<=$J$12	Binding	0
$H$13	Insole Stitching Time (minutes) Used	840	$H$13<=$J$13	Binding	0
$H$14	Molding Time (minutes) Used	2100	$H$14<=$J$14	Binding	0
$H$15	Sole-to-Upper Joining Time (minutes) Used	2100	$H$15<=$J$15	Binding	0
$H$16	Inspecting Time (minutes) Used	798.62693	$H$16<=$J$16	Not Binding	41.37307002

Figure 2.3.6: Answer Report for Pallas Sport Shoe Company with 415-minute Cutting Time constraint

Q8. How does the Answer Report in Figure 2.3.6 differ from the one in Figure 2.3.4?

Q9. Is the first constraint still binding? Do you think Pallas Sport Shoe Company should subtract this five minutes of cutting time each day? Explain your reasoning.

Now, suppose Pallas Sport Shoe Company considers adding another 130 minutes of cutting time each day. Therefore, the cutting time constraint is changed from 420 to 550. The Answer Report for this new scenario is shown in Figure 2.3.7.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$J$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,781.57

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [daily production rate] High Rise (x1)	0	0	Contin
$C$6	Decision Values [daily production rate] Max-Riser (x2)	0	61.57068063	Contin
$D$6	Decision Values [daily production rate] Stuff It (x3)	0	131.9371728	Contin
$E$6	Decision Values [daily production rate] Zoom (x4)	0	123.1413613	Contin
$F$6	Decision Values [daily production rate] Sprint (x5)	0	0	Contin
$G$6	Decision Values [daily production rate] Rocket (x6)	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$11	Cutting Time (minutes) Used	536.5445026	$H$11<=$J$11	Not Binding	13.45549738
$H$12	Upper Finishing Time (minutes) Used	1260	$H$12<=$J$12	Binding	0
$H$13	Insole Stitching Time (minutes) Used	840	$H$13<=$J$13	Binding	0
$H$14	Molding Time (minutes) Used	2093.403141	$H$14<=$J$14	Not Binding	6.596858639
$H$15	Sole-to-Upper Joining Time (minutes) Used	2100	$H$15<=$J$15	Binding	0
$H$16	Inspecting Time (minutes) Used	818.0104712	$H$16<=$J$16	Not Binding	21.9895288

Figure 2.3.7: Answer Report for Pallas Sport Shoe Company with 550-minute Cutting Time constraint

Q10. How does the Answer Report in Figure 2.3.7 differ from the one in Figure 2.3.4?

Q11. Do you think Pallas Sport Shoe Company should add this extra 130 minutes of cutting time each day? Explain your reasoning.

Recall that the decision variable x_1 is not in the optimal solution. A logical question to ask is whether increasing the profitability of x_1 could allow it to enter the optimal solution and, if so, how much of an increase would be necessary. Return again to the spreadsheet in Figure 2.3.2, reset the cutting time constraint to its original value of 420 minutes, and change the value of the objective function coefficient of x_1 from 18 to 19. The Answer Report for this new scenario is shown in Figure 2.3.8.

Q12. In terms of the problem, what does this change represent?

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$J$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,216.90

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [daily production rate] High Rise (x1)	0	170.6951872	Contin
$C$6	Decision Values [daily production rate] Max-Riser (x2)	0	8.983957219	Contin
$D$6	Decision Values [daily production rate] Stuff It (x3)	0	125.7754011	Contin
$E$6	Decision Values [daily production rate] Zoom (x4)	0	0	Contin
$F$6	Decision Values [daily production rate] Sprint (x5)	0	0	Contin
$G$6	Decision Values [daily production rate] Rocket (x6)	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$11	Cutting Time (minutes) Used	420	$H$11<=$J$11	Binding	0
$H$12	Upper Finishing Time (minutes) Used	1260	$H$12<=$J$12	Binding	0
$H$13	Insole Stitching Time (minutes) Used	716.4705882	$H$13<=$J$13	Not Binding	123.5294118
$H$14	Molding Time (minutes) Used	1873.15508	$H$14<=$J$14	Not Binding	226.8449198
$H$15	Sole-to-Upper Joining Time (minutes) Used	2100	$H$15<=$J$15	Binding	0
$H$16	Inspecting Time (minutes) Used	619.8930481	$H$16<=$J$16	Not Binding	220.1069519

Figure 2.3.8: Answer Report for Pallas Sport Shoe Company with $19 High Rise shoe profit

Q13. How does the Answer Report in Figure 2.3.8 differ from the one in Figure 2.3.4?

Q14. Why may Pallas Sport Shoe Company not want this new optimal solution?

Q15. Explain, in your own words, the usefulness of Answer Reports.

Section 2.4: Chapter 2 (LP Maximization) Homework Questions

1. Anderson Cell Phone Company has started cell phone production. It produces smart phones and standard phones. Initially, they hired 10 workers for the assembly line. The workers are paid for 8 hours per day. However, they spend only 7 hours assembling the phones because of a 30-minute lunch break and two 15-minute breaks. A smart phone takes 2.5 minutes to assemble and a standard phone takes 1.5 minutes to assemble. The company receives a delivery of 2000 LCD screens per day from its supplier. Profit margins for a smart phone and a standard phone are $40 and $30, respectively. Anderson Company is interested in determining the product mix that gives them the highest daily profit.

   a. Define the decision variables for the problem.

   b. Use the decision variables to define the objective function.

   c. Use decision variables to define the assembly constraint.

   d. Use decision variables to define the screen constraint.

   e. Graph the feasible region.

   f. Pick two points in the feasible region and calculate the profit for each point.

   g. What are the corner points of the feasible region?

   h. Calculate the profit at each corner point.

   i. Which point gives the highest profit? What is the optimal product mix?

   j. Find the optimal production plan with Excel Solver to verify the optimal solution.

2. GA Sports makes professional and regular soccer balls. Typically, soccer balls are made up of 4 elements: the cover, the stitching, the lining, and the bladder. GA Sports uses a synthetic leather cover for professional balls and puts 4 layers of cotton linings underneath. It uses a rubber cover for regular balls and puts 3 layers of cotton linings underneath. The cover and the linings are created by cutting large pieces of each material into 32 panels for a professional soccer ball and 18 panels for a regular soccer ball. It takes 14 minutes to carefully cut the casing for a professional ball. It takes only 7 minutes to cut the material for a regular ball. GA Sports has one experienced cutter who works 7 hours per day. The 32 panels of a professional soccer ball are stitched together. An experienced worker can stitch 4 balls in a workday. The company has four experienced stitchers. The regular balls are thermally molded on a special machine in twelve minutes. The company has one machine that is run by a machinist for six hours per day. The company makes $15 profit on professional balls and $10 on regular balls.

   a. Define the decision variables for the problem.

   b. Use the decision variables to define the objective function.

   c. Use decision variables to define the cutting time constraint.

   d. Use decision variables to define the stitching time constraint.

   e. Formulate the problem.

   f. Find the feasible region graphically.

   g. Find the optimal production plan using Excel Solver.

3. Family Cow is a small dairy. The owner is the only person who works in the dairy; he works 8 hours per day. He makes and sells cream and butter and donates the skim milk to a charity nearby. He is able to make 1 quart of cream from 10 quarts of milk or ½ pound of butter from 10 quarts of milk. It takes 20 minutes to get a quart of cream and 30 minutes to make butter. The dairy has 300 quarts of milk daily. Family Cow's prices are higher because the products are organic: a quart of organic cream is sold for $7 and a pound of organic butter for $12. He can only sell eight pounds of butter per day to a local store.

    a. Define the decision variables for the problem.

    b. Use the decision variables to define the objective function.

    c. Formulate the problem.

    d. Determine the optimal solution graphically.

    e. Find the optimal production plan using Excel Solver.

4. The owner of The Family Cow dairy has been using the optimal production plan for several months. He is considering adding plain yogurt and cheese to the products he sells. It takes 10 quarts of milk to produce 7 lbs of yogurt or 3 lbs of cheese. Prices of yogurt and cheese are $3.5/lb and $5.5/lb, respectively. It takes 10 minutes to make a pound of yogurt and 15 minutes for a pound of cheese.

    a. Define the additional decision variables.

    b. Formulate the problem.

    c. Find the new optimal product mix and the daily profit using Excel Solver. How many different products does he make? How much more money will he make by offering two more products? Would you recommend he change his production plan?

5. Katia has won $200,000 from the lottery. She considers investing the money in a bond fund and a domestic stock fund. The projected annual return for the bond fund is 8% and for the stock fund is 15%. Her friend is experienced with investments. She suggests she invest at most $75,000 in the stock fund. However, the stock and bond funds could go down in value. The brokerage firm told Katia that the most she could lose in the next year is 5% of her original investment in the bond fund and 20% of her original investment in the stock fund. She wants to limit her total potential losses to no more than $20,000. How should she invest her winnings?

    a. Define the decision variables for the problem.

    b. Formulate the problem.

    c. Determine the optimal investment plan using Excel Solver.

6. In addition to the bond fund and the domestic stock fund, Katia also considers investing in an international stock fund. The projected annual return for that stock fund is 22%. The maximum investment in any stocks is still $75,000. The most she could lose in the next year in this international stock is 25%.

    a. What is her optimal investment strategy?

    b. Will she make more money than before?

7. John Farmer is studying operations research in school. He is curious about applying what he learns in class to actual problems. John's father owns a farm in Missouri and has 640 acres under cultivation. John would like to help his father determine the mix of corn and soybeans to plant that would maximize his father's profit. Table 2.4.1 contains some data that John collected about corn and soybean production in Missouri.

	Corn	**Soybeans**
Price/Bushel	$2.90-3.50	$7.85-8.85
Yield/Acre	155.8 bu.	41.4 bu.
Seed Cost/Acre	$45.50	$34.00
Fertilizer cost/acre	$88.10	$37.80
Fuel Cost/Acre	$24.40	$10.00
Worker Cost/Acre	$13.00	$9.50

Table 2.4.1: 2007-08 USDA corn and soybean estimates for Missouri

a.) In order to formulate the problem, John must know the price per bushel at which corn and soybeans can be sold. However, the USDA data contains a range of values for both of these crops. What value do you think John should use for each? Why?

b.) What is the largest gross revenue before considering expenses that Mr. Farmer can make? What crop mix produces this gross revenue?

c.) What is the largest net revenue after expenses that Mr. Farmer can make? What crop mix produces this net revenue?

d.) Suppose Mr. Farmer has budgeted only $60,000 to cover all of the expenses of his crop production. What is the largest net revenue he can earn under this constraint?

8. Corn and soybeans are used in the production of biofuels. Biofuel consumption is important for the environment, because greenhouse gas emissions are reduced 12% by ethanol combustion and 41% by biodiesel combustion. The total corn and soybean production in the United States can meet only 12% of the demand for gasoline and 6% of the demand for diesel fuel. In order to encourage corn production, the USDA pays a subsidy to increase the price per bushel to $3.50. However, to get the subsidy, the farmer must produce at least 40,000 bushels of corn.

   Assuming that the price per bushel that Mr. Farmer can sell his corn for without the subsidy is $2.90, should Mr. Farmer accept the constraint of producing at least 40,000 bushels of corn in order to earn the subsidy?

9. After doing some research, John learned that soybean followed by corn a year after, increases corn yield by 7.5% and saves 25% of the soybean residue nitrogen, which will reduce the fertilizer use in corn production by $2.50 per acre. The reduction in fertilizer use will also reduce its harmful effect to the environment. In the news, he heard that nitrate leaching causes surface and ground water to degrade. This will harm the living things that use water for drinking or swimming. For all of these reasons, John wants to convince his father, who planted 200 acres of land with soybean last year, to begin to rotate corn and soybeans.

   Should John's father begin to rotate his corn and soybean crops?

10. John gathered some data related to wheat production (Table 2.4.2).

    Is it profitable to produce some wheat under the constraints in question 7?

	Corn	Soybean	Wheat
Price/bushel	$2.90-3.50	$7.85-8.85	$3.75-4.15
Yield/acre	155.8 bushels	41.4 bushels	60 bushels
Seed cost/acre	$45.50	$34	$24
Fertilizer cost/acre	$88.10	$37.80	$69.25
Fuel cost/acre	$24.40	$10	$40
Worker cost/acre	$13	$9.50	$9

Table 2.4.2: 2007-2008 Corn, soybean, and wheat estimates for Missouri

11. At the end of Problem 2.1, you were asked to formulate the Computer Flips problem after the addition of two new products. Your formulation should have used four decision variables and should have included eight constraints, including four non-negativity constraints. In an Excel spreadsheet, set up this formulation, and use solver to obtain a solution.

    a. What is the optimal solution?

    b. What does this optimal product mix imply about the planned addition of new products?

    c. What research do you think the Sales Department should conduct before implementing the optimal product mix?

12. Elegant Fragrances, Ltd. decides to produce two new perfumes, *L'Arbre d'Amour* and *Evening Rose.* The factory management asked the industrial engineering department to develop a mathematical model for maximizing the profit obtained from producing these products. There are also some limitations in resources, budget and the capacity of the factory that should be considered in the model. At the beginning, the industrial engineering team studied the problem and gathered information from which a model can be developed. The team collected the following information:
    - Each perfume is made of two main components. A fragrant perfume oil and a solvent. The solvent, such as a combination of ethanol and water, is necessary to reduce the allergic reactions of skin to the perfume oil. The solvent is a large percentage of the overall final product.
    - A fragrant oil for *L'Arbre d'Amour* is obtained from Mango Pulp, Tea Leaves, and Juniper Berry, and a fragrant oil for *Evening Rose* is obtained from Mango Pulp, Tea Leaves, and White Rose.
    - There are two main processes in the production of these perfumes: extraction and blending. In the extraction stage, physical and chemical processes change the raw materials and the perfume oil is extracted. In the blending stage, the perfume oil is blended with the solvent. However, Elegant Fragrances does not work directly with the fragrance raw materials, but instead purchases fragrance essences from suppliers and only blends them.

			L'Arbre d'Amou	***Evening Rose***
Income (per pound)			$370	$215
Percentage of raw materials	**Fragrance Oil (Pure)**	**Mango**	3%	5%
		Tea Leaves	5%	7%
		Juniper Berry	7%	0
		White Rose	0	8%
	Solvent	**Ethanol and Water**	85%	80%
Process Costs (Per Pound)		**Blending**	$12	$12

Table 2.4.3: Some information about the perfumes

	Availability (pounds per year)	**Cost ($ per pounds)**
Mango	1400	14
Tea Leaves	1600	10
Juniper Berry	1000	384
White Rose	1500	56
Ethanol and Water	Unlimited	7

Table 2.4.4: Ingredient information

The total cost for making each perfume is the $12 processing cost plus the cost of ingredients. The cost for ingredients to make ***L'Arbre d'Amou*** is determined by multiplying the percentage of each ingredient by the corresponding costs.

Ingredient cost = .03(14) + .05(10) + .07(384) + 0(56) + .85(7) = $33.75

Net Profit = 370.00 – 12 – 33.75 = $324.25

a. Determine the net profit for ***Evening Rose***.

b. Define the decision variables as the amount of annual production of each perfume and help the industrial engineering team formulate this problem for maximizing the profit within the given constraints.

c. Use Solver to obtain the optimal solution for the Elegant Fragrances problem.

13. The management at Elegant Fragrances is considering producing two new perfumes, *Evergreen* and *Embrasser du Soir.* The income and costs, as well as the key ingredients and their proportions for the new perfumes are given in Table 5 below. If the availability of resources is unchanged, what are the optimal production rates for each of the four perfumes so as to maximize profit?

			Evergreen	***Embrasser du Soir***
Income (per pound)			$490	$235
Percentage of raw materials	**Fragrance Oil (Pure)**	**Mango**	8%	7%
		Tea Leaves	7%	5%
		Juniper Berry	9%	0
		White Rose	0	10%
	Solvent	**Ethanol and Water**	76%	78%
Process Costs (Per Pound)		**Blending**	$12	$12

Table 2.4.5: New Perfumes Information

Chapter 2 Summary

What have we learned?

Linear programming is a process of taking a real world situation, modeling it with inequalities, and finding the best or optimal solution.

- Modeling the situation
  - We start by finding the decision variables – what things can you choose? Typically this is how much to make of a particular product or how much to invest in a particular option. We assign a variable for each of these choices.
  - Next we write an objective function that captures the goal of the problem. – What will determine when you have found the optimal solution? This is the equation that we want to maximize.
  - Finally we define the constraints - those things that limit our choices. These are typically the amount of money, time, people, or resources available.

- Once we have defined our problem, we use a spreadsheet program such as Microsoft Excel to find the optimal solution. After entering the inequalities we set up the solver parameters and run solver. This gives us an answer and sensitivity report.

- The answer report will show:
  - The objective function's final value
  - The value for each of the decision variables
  - The amount of each constraint that is used

- Last we perform sensitivity analysis to determine the effect of different changes.
  - What if the situation changes? Will the optimal solution change and if so by how much?
  - How much would the situation have to change before we would need to re-run solver?

Terms

Adjustable Cells	A column in the Sensitivity Report that shows the increase/decrease of an objective function coefficient without changing the final values and only applies to one objective function coefficient at a time (all other coefficients must remain constant)
Allowable Decrease	A column in the Sensitivity Report that tells how much you can *decrease* the objective coefficient without changing the final values; this decrease will cause the optimal total profit to decrease by an unknown amount
Allowable Increase	A column in the Sensitivity Report that tells how much you can *increase* the objective coefficient without changing the final values; this increase will cause the optimal total profit to increase by an unknown amount
Answer Report	A report that details the optimal solution, lists whether constraints are binding or non-binding, and gives the slack for each constraint
Binding Constraint	A constraint that is satisfied as a strict equality in the optimal solution; all of the available constraint is used
Constraint	A condition that must be satisfied, represented by equations or inequalities
Decision Variable	A quantity that the decision-maker controls
Feasible Solution	A solution that satisfies all the constraints
Final Value	A column in the Answer and Sensitivity Reports that refers to the number in the decision variable cells *after* you use Solver (i.e., the decision variable and objective function values for the optimal solution)
Line of Constant Profit	A line representing the objective function, where every point on the line generates the same profit
Linear Programming	A mathematical technique for finding the optimal value of a linear objective function subject to linear constraints when the decision variables can take on fractional values.
Mathematical Programming	A mathematical approach to allocating limited resources among options in an optimal manner (includes linear programming, integer programming, and binary programming)
Non-Binding Constraint	A constraint for which all available resources are not used
Objective Function	The function that is to be optimized

Optimal Solution	The feasible solution with the best value for the objective function
Original Value	A column in the Answer Report that refers to the number in the decision variable cells *before* you use Solver
Product Mix	The composition of all goods and/or services being produced
Production Rate	The number of products made in a given period of time
Reduced Cost	A column in the Sensitivity Report that shows how much an objective function's coefficient would have to change in order to change the optimal mix; in other words, reduced cost is the per-unit amount that the product contributes to profits, minus the shadow price (note: if reduced cost is negative, then the product is not profitable to make)
Sensitivity Report	A report that shows how changes to the situation will affect the optimal solution
Shadow Price	A column in the Sensitivity Report that gives the amount the objective function would change if there is a *one unit* change in the right-hand side of a constraint
Slack	The amount of a resource that is not used in the optimal solution

Chapter 2 (LP Maximization) Objectives

You should be able to:

- Identify the decision variables
- Define the objective function by finding the goal to be solved for the situation
- Identify the constraints and write inequalities to model them.
- Enter each of these into Microsoft Excel
- Use solver to find the Optimal Solution and generate Answer and Sensitivity Reports
- Analyze the Answer Report
- Analyze the Sensitivity Report

Troubleshooting

What can go wrong?

Troubleshooting is a valuable skill when using Excel and Solver. Quite often a problem is developed and solved and the answer will not make sense. As you work through this book try to remember the mistakes you make so you can avoid them in the future.

- One thing that students may do when working through the problems with the text is to look ahead. They realize that the answers to the questions are on the next page and will copy them onto their spreadsheet. This can lead to problems running solver. When setting up the constraints, the right side must be a value – how much of the resource is available. The left side must be a formula that multiplies the decision variables by the amount used for each one. This formula does not appear on the screen, only the value appears. Typing values printed in the book rather than the formula will cause a problem.
- Another common mistake is to forget to change the direction of the inequality in solver. The default in solver is a less than or equal (<=) constraint. For example, if you have up to 40 hours to produce something, you use the <= constraint. Some situations require something to be at least a certain value. For example if you make tables and chairs you have to make at least 4 chairs for each table. This requires you to use the greater than or equal (>=) constraint.
- When a model changes, don't forget to make the changes in solver. Recall that when we develop a spreadsheet, we start by entering our decision variables, then write our objective function, and finally a row for each constraint. We then start solver and enter the decision variables, objective function and constraints. If you revise a problem, either by adding new decision variables or adding new constraints, you follow the same process. First change the decision variables and/or constraints in excel but then you must make the same changes in solver. Otherwise the computer will not recognize that the situation has changed.

Chapter 2 Study Guide

1. What are decision variables? Where do they come from in the word problem?

2. What is the objective function? Where does it come from in the word problem?

3. What are constraints? Where do they come from in the word problem?

References

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Appendix A: Using Excel Solver in Microsoft Office 2003 and 2007

The majority of the steps for using Excel Solver are the same in all versions of Microsoft Office. However, there a few differences.

Microsoft Office 2003

To add in Solver in Microsoft Office 2003, go to Add-Ins under the Tools menu and click on it. The Add-Ins window will appear. Then, check the Solver Add-In box and then click OK.

Next, set up the spreadsheet in the same way as detailed in Steps 1-4 of Section 2.2.6.

Once the spreadsheet is properly set up, click on the cell containing the objective function and choose Solver from the Tools menu. A Solver Parameters window will open, as shown in Figure 2.A.1.

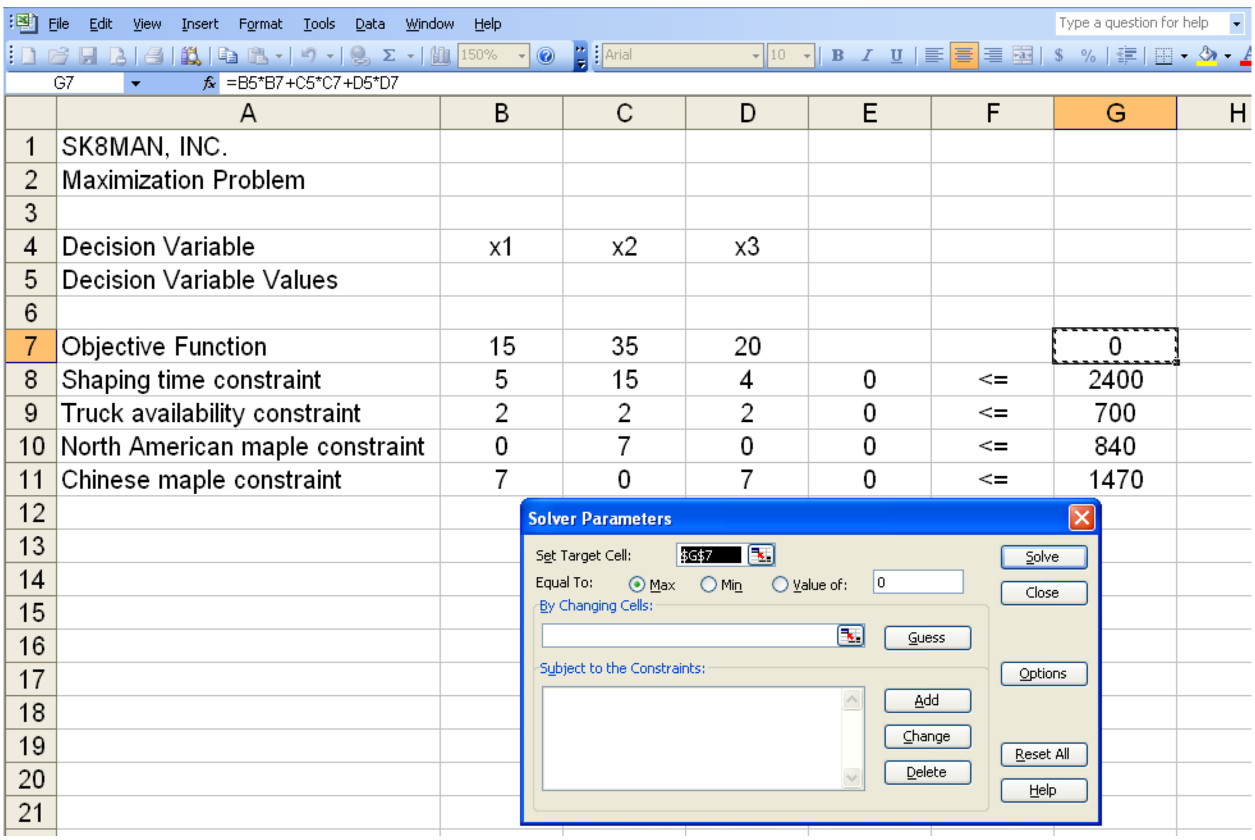

Figure 2.A.1: Solver Parameters window in Microsoft Office 2003

Verify that the "Max" circle is filled in. Then fill in the "By Changing Cells" and "Subject to the Constraints" windows as described in Steps 7-8 in Section 2.2.6

To include the non-negativity constraint, choose Options and check the box that says "Assume Non-Negative." Also, check the box that says "Assume Linear Model." See Figure 2.A.2 for an illustration of this.

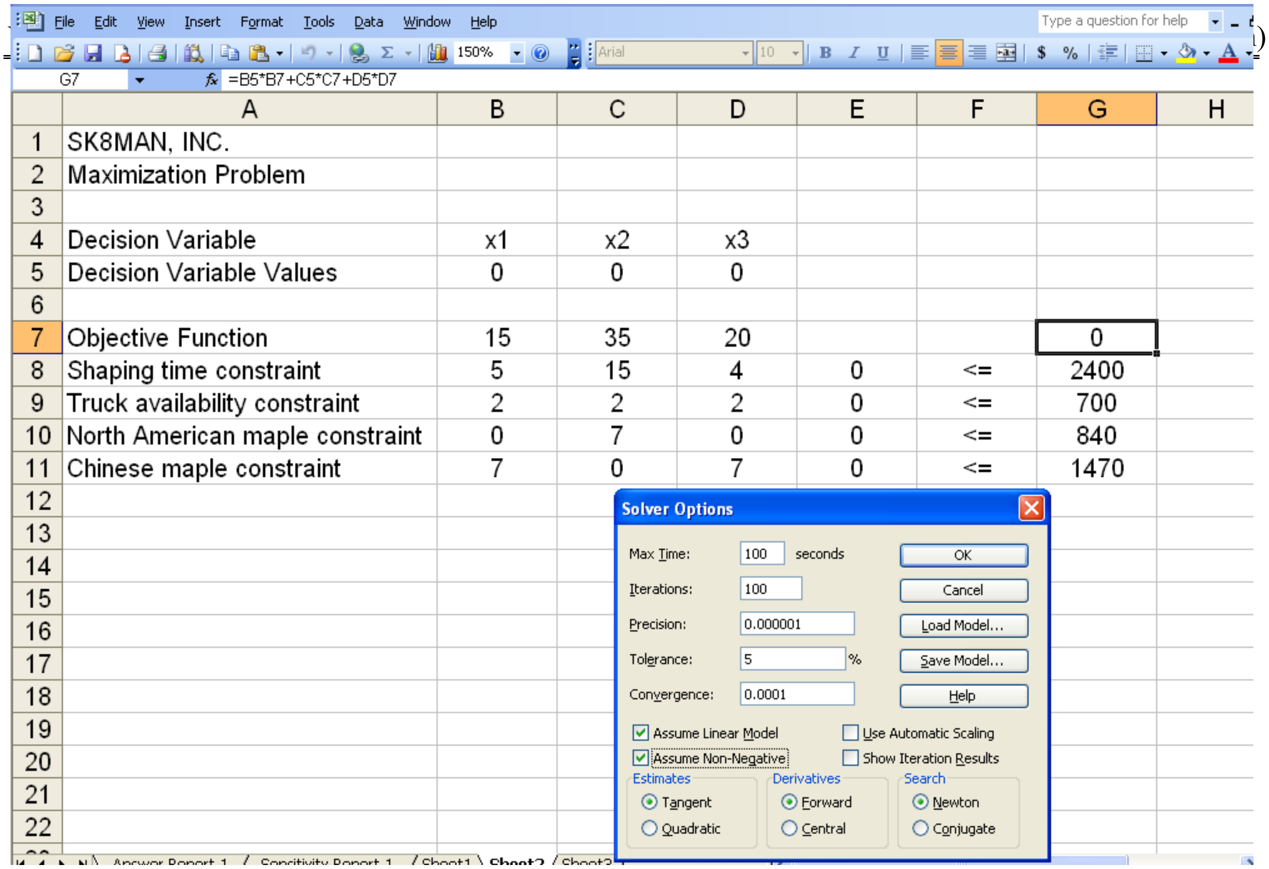

Figure 2.A.2: Solver Options window in Microsoft 2003

Finally, click "Solve" and choose the desired reports, as shown in Figure 2.A.3.

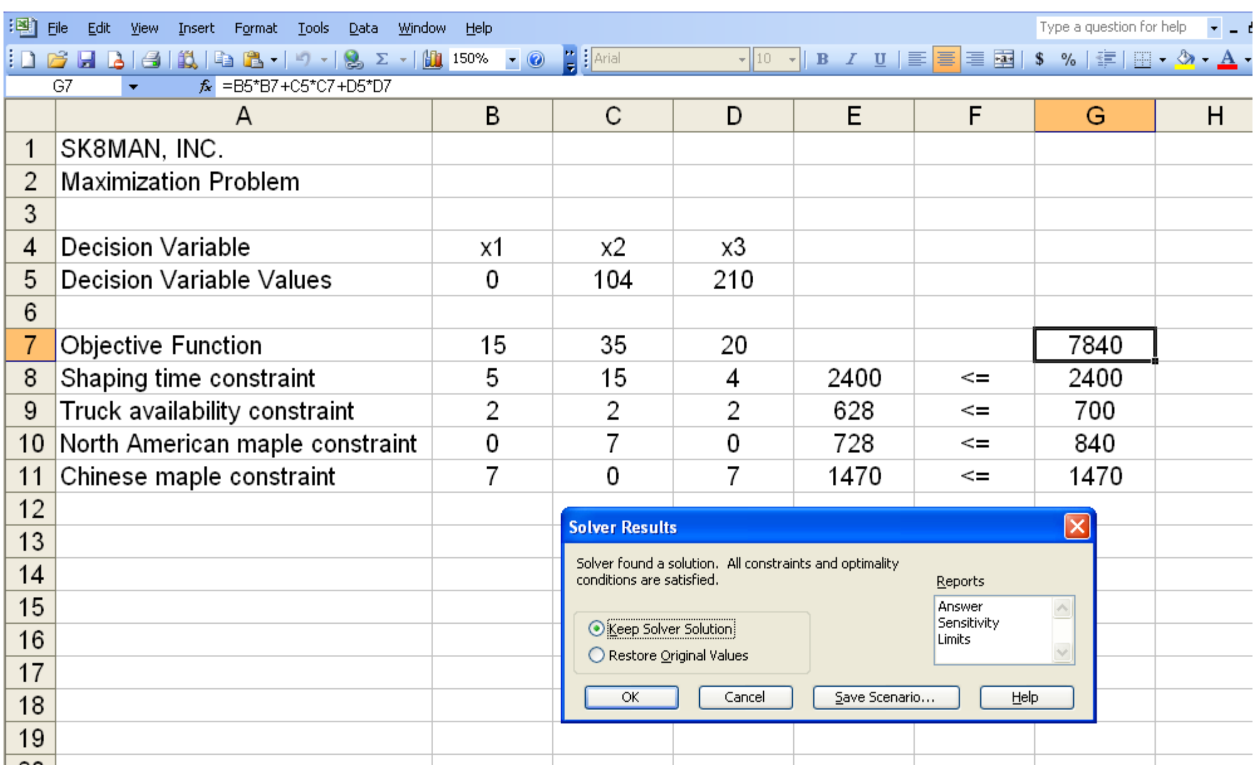

Figure 2.A.3: Solver results window

Microsoft Office 2007

To add in Solver in Microsoft Office 2007, click the Microsoft Office Button and choose “Excel Options.” Click “Add-Ins.” Under the “Manage” box, select “Excel Add-ins” and click “Go.” Check the “Solver Add-in” box and click “OK.”

Once Solver has been added, the Solver command is in the Analysis group on the Data tab, as shown in Figure 2.2.9.

All other steps for using Excel Solver in Microsoft Office 2007 are the same as for Microsoft Office 2003, given above.