Section 10.0: Introduction

We all make decisions in our jobs, our communities and in our personal lives that involve significant uncertainty. Some examples are:

- How much technology should be invested in a plant when product demand is uncertain?
- How much car collision insurance should be purchased when you do not know whether or not you will have an accident?
- Should a company build a new power plant in a foreign country and, if yes, in which country?
- On a personal financial level, how much should you invest in a particular stock or mutual fund?
- Should millions be invested in a new drug that has proven effective in animal tests?
- How much time should you invest in studying when the questions on the exam are uncertain and payback from more study-time is hard to predict?
- What career should you pursue when the economy and job market are uncertain?

Specific stocks and the stock market as a whole demonstrate much uncertainty from day to day and year to year. Individuals experience different types of random accidents. There is uncertainty in the demand for power and the stability of developing countries. The link between animal drug trials and drug effectiveness in humans is far from perfect. Decision analysis is an operations research modeling tool used to select the best decision in the presence of uncertainty.

- What specific uncertainties do you face in the next day, week, or month?
- What about other members of your family or friends?
- What uncertainties will affect your planning over the next year?

The oil industry was one of the earliest users of decision analysis and continues to lead in its application. Pharmaceutical companies apply decision trees, which are an extension of probability trees, to make research and development decisions. Industrial giants such as DuPont, Xerox, and Kodak have used decision trees to plan new products and production capacity. The US Forest Service uses decision trees to plan controlled forest fires. The Decision Analysis Affinity Group (www.daag.org) is an organization that runs conferences at which corporate users of decision analysis share experiences. There is also the Decision Analysis Society which is an organization affiliated with INFORMS. They maintain a homepage at http://faculty.fuqua.duke.edu/daweb/.

The decision tree methodology involves accounting for every possible decision and outcome. The best alternative generally maximizes the expected value of profit or minimizes the expected value of cost. Sometimes the variable optimized is not financial. The goal might be minimize time or maximize the number of people attending an event. Modern software such as Precision Tree, an Excel add-on, helps with the analysis and offers graphical representations of the results. These enable a decision maker to explore the strengths and weaknesses of the alternatives.

As decision analysis developed, the leaders in the field recognized two critical psychological and practical issues that needed to be addressed in order to make the tool of greater practical value. First, the models required estimates of probabilities that were often not easy to obtain with a detailed analysis of data. Therefore, subject matter experts were interviewed in order to estimate the probabilities. Decision analysts, along with mathematical psychologists, became leaders in the effort to understand biases and misconceptions that individuals display when asked to make a forecast. They developed interview protocols in order to get expert opinion in a way that reduces the likely bias.

Second, the expected value does not capture the fact that people are often fearful of taking risks, especially large ones. This risk aversion is the foundation for all of the insurance industry and the huge market in extended warranties. Decision analysts became leaders in researching attitudes towards risk and designing a methodology called utility theory that captures this behavior.

Section 10.1: Planning a Wedding

Chris and Mary Ann are planning their own wedding. They have hired a wedding planner to help them make the arrangements. Chris and Mary Ann have determined that they can afford to have no more than 240 people attend their wedding reception. They have decided that their main goal is to maximize the number of people attending their reception without going over their limit of 240.

As is usually done, Chris and Mary Ann will mail out wedding invitations requesting an RSVP indicating the invitee's acceptance of the invitation or not. Their wedding planner, Amy Smith, tells them that based on her experience, the probability that one-half of the invitees will accept the invitation is 0.2, the probability that two-thirds will accept is 0.5, and the probability that three-fourths will accept is 0.3. Ms. Smith says that although these probabilities are estimates, they reflect what she has seen in her 25 years as a wedding planner.

Furthermore, Ms. Smith knows from experience that there will always be "no shows", people who accept the invitation, but for some reason do not attend. Based on her experience, she estimates that the probability of a 5% rate of no shows is 0.6 and the probability of a 10% no show rate is 0.4. She again explains to Mary Ann and Chris that these probabilities are estimates based on her many years of experience.

10.1.1 How Many People Should We Invite?

Chris and Mary Ann decide to explore what might happen if they invite 240 people. They realize that Ms. Smith's estimates lead to multiple possibilities, and that each possible outcome is composed of two parts. They decide to diagram the situation to help them better understand it. Their diagram appears in Figure 10.1.1.

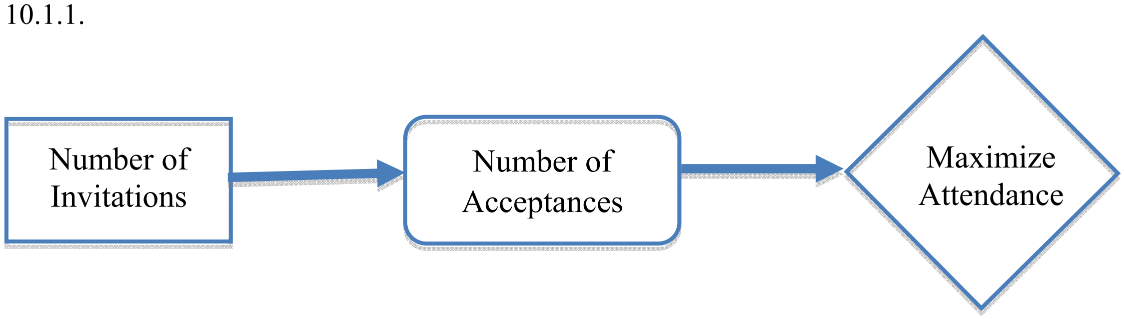

Figure 10.1.1: Chris and Mary Ann diagram their wedding scenario.

It seems to Chris and Mary Ann that something is missing from their diagram. Then Ms. Smith points out that her estimates of the percentages of acceptances and no shows do not appear in the diagram.

Q1. Which box or boxes in the diagram are affected by the percentage of people invited who accept the invitation?

Q2. Which box or boxes in the diagram are affected by the percentage of no shows?

After discussing the situation with Ms. Smith, Chris and Mary Ann revise their diagram by adding two uncertain events. Their new diagram is shown in Figure 10.1.2.

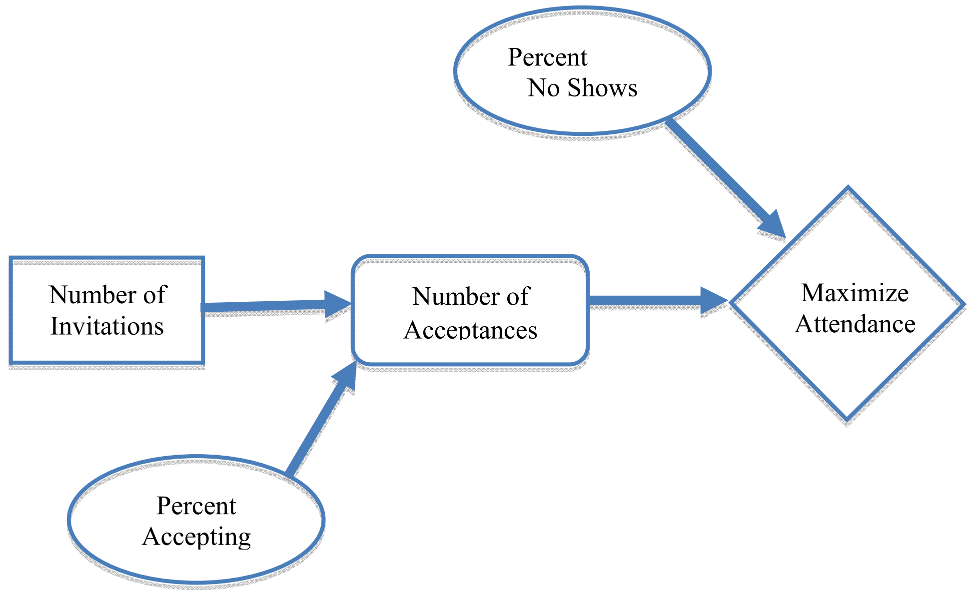

Figure 10.1.2: The wedding scenario showing rates of acceptance and no shows

After analyzing their revised diagram, Chris and Mary Ann can see two factors that will affect the number of people they invite who attend their wedding reception: the per cent who accept their invitation and the per cent of no shows. Next, they try to determine how many possibilities there would be if they used Ms. Smith's estimates, but are unable to agree.

Mary Ann says that she thinks there are six, but Chris is skeptical. Mary Ann explains her thinking this way:

> First, Amy Smith gave us three possible rates of acceptance. Then she gave us two possible percentages of no shows. The two no show percentages would apply to each of the three acceptance rates. So that would be six possibilities. For example, the acceptance rate might be one-half, and the no show percentage might be 5%.

Then Ms. Smith offers to draw a diagram of Mary Ann's idea (See Figure 10.1.3).

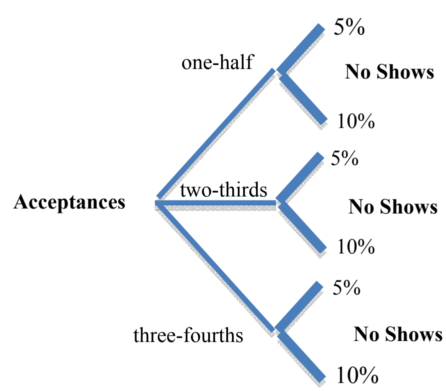

Figure 10.1.3: Ms. Smith's diagram showing six possibilities

Q3. Identify the portion of the diagram in Figure 10.1.3 that represents an acceptance rate of one-half and 5% no shows.

Q4. List the other five possibilities, based on Amy Smith's estimates.

Mary Ann applied the **Fundamental Principle of Counting** to determine how many possibilities there are in this scenario. This principle states that if there are 3 possible outcomes for one event (e.g., the percentage of acceptances) and 2 possible outcomes for a second event (e.g., the rate of no shows), then there are $3 \cdot 2 = 6$ ways in which both events might occur. More generally,

> **The Fundamental Principle of Counting**: If there are m possible outcomes for one event and n possible outcomes for a second event, then there are mn possible ways in which both events can occur.

The diagram that Amy Smith drew to clarify Mary Ann's thinking is called a **tree diagram**. A tree diagram is useful for identifying all of the possibilities when several things occur. It is also a useful tool when determining the probability of a **compound event**; that is, an event that is the result of more than one outcome. For example, the event that the acceptance rate will be one-half *and* the percentage of no shows will be 5% is a compound event, because it consists of two separate outcomes.

Once Chris and Mary Ann agree on the number of outcomes, they begin to wonder how many people are likely to attend. This time Chris takes the lead. He reasons that if one-half of the people who are invited accept the invitation, that's 120 people. Then if 5% do not show up, 5% of 120 is 6 no shows. Finally, if there are 6 no shows out of the 120 people who accepted the invitation, then 114 people will actually attend.

Q5. Do you think that Chris's reasoning is sound? Why/why not?

Q6. Complete a table similar to the one below to estimate the number of people who attend Chris and Mary Ann's wedding reception for each of the 6 possible outcomes that they identified based on Amy Smith's estimates.

Acceptance rate	**Percentage of no shows**	**Number of people likely to attend**
One-half	5%	(240)(0.5)(0.95)=114
r	p	

Table 10.1.1: Estimating Attendance for Six Possible Outcomes.

Now Chris and Mary Ann decide that they need to know the average number of people who will attend. They have calculated a number for each of 6 possible outcomes, but now they wonder if they can just average those six numbers. Chris says that he thinks they can, *if* the 6 possible outcomes are equally likely to occur.

Q7. Do you think that Chris is right about averaging the six numbers if they are equally likely? Why/why not?

Q8. Do you think that the 6 possible outcomes are equally likely?

Mary Ann then says that she thinks she knows how they can determine the probability of each of the 6 possible outcomes. She argues that if the probability that only one-half of the people invited is 0.2, that's the same as $\frac{2}{10}$. Then, if the probability that there will be 5% no shows is 0.6, that's like $\frac{6}{10}$. Finally, she says that they need to find $\frac{6}{10}$ of $\frac{2}{10}$, and that would be $\frac{12}{100}$.

Q9. How did Mary Ann come up with $\frac{12}{100}$?

Q10. Do you think Mary Ann's argument is correct?

Now Chris and Mary Ann wonder what a probability of $\frac{12}{100}$ means for their wedding plans. Amy Smith, their wedding planner, says it means that out of every 100 weddings that she has planned, on average there were 12 times when the acceptance rate was one-half and the percentage of no shows was 5%. So, in terms of Mary Ann and Chris's wedding, there are 12 chances in every hundred that 114 of the 240 invitees will actually attend. Then Amy adds her probability estimates to the tree diagram she drew earlier. She tells Mary Ann and Chris that the revised diagram will help them to compute the probabilities of the other five possibilities.

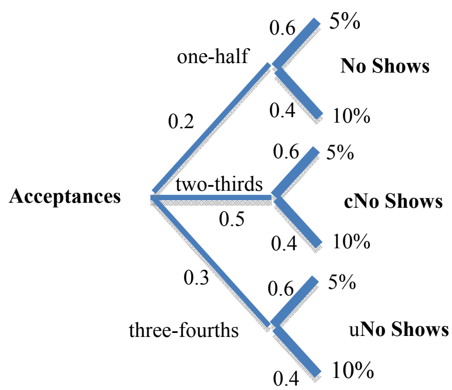

Figure 10.1.4: Amy Smith's tree diagram with her probability estimates

Q11. Complete a table similar to the one below by finding the probability for each of the 6 possible outcomes.

Acceptance rate	% of no shows	Likely # attending	Probability
One-half	5%	114	0.12
One-half	10%		
Two-thirds			
Two-thirds			
Three-fourths			
Three-fourths			

Table 10.1.2: Finding the Probability of Each of the Six Possible Outcomes

Q12. Which of the 6 possible outcomes is the most likely to occur? How many people would attend if that possibility occurred?

Q13. Which of the 6 possible outcomes is the least likely to occur? How many people would attend if that possibility occurred?

Q14. What is the sum of the six probabilities in your table? Why does that make sense (or not)?

Q15. What is the sum of the two probabilities for which the acceptance rate was one-half? … two-thirds? … three-fourths? Why do these sums make sense (or not)?

Q16. Do results similar to those in question 15 occur when you consider the three probabilities for which the percentages of no shows were 5% and 10%?

Q17. Using the results in your table, what is the probability that the likely number of people attending will be less than 110? … greater than or equal to 110?

Q18. Using the results in your table, what is the probability that the likely number of people attending will be less than 120? … greater than or equal to 120?

Q19. Using the results in your table, what is the probability that the likely number of people attending will be less than 150? … greater than or equal to 150?

When you answer these questions, you may find it helpful to sort the possible outcomes from the lowest to highest number of attendees. This has been done in Table 10.1.3

Acceptance rate	% of no shows	Likely # attending	Probability	Cumulative Probability
One-half	10%			
One-half	5%	114	0.12	.20
Two-thirds				
Two-thirds				
Three-fourths				
Three-fourths				

Table 10.1.3: Ordering the outcomes by the number attending

The method that Mary Ann used to find the probability that one-half of the invitees will accept the invitation and that there will be 5% no shows is called the **Multiplication Rule for Compound Events.** In order to apply the *multiplication rule* to a *compound* event, all of the separate outcomes that make up the compound event must be **independent**. Independent events are events for which the probabilities of the outcomes do not affect one another. For example, consider the following.

Every year medicine makes progress in understanding the role of genetics in medical disorders. It has long been known that some genetic defects are independent of the gender of the individual and others are linked to the gender of the individual. For example, cystic fibrosis occurs equally in males and females. In contrast, color blindness is much more common amongst males than females. In the United States, an estimated 7% of the males have color blindness and only 0.4% of the females. If a name is selected at random, the event that the individual is a male and the event that the individual has cystic fibrosis are independent. In contrast, if a name is selected at random, the event that the individual is a male and the event that the individual is color-blind are not independent. In other words, once you identify the individual as male, your estimate of the probability of color blindness increases significantly.

As a second example of events that are independent or not, suppose that, on the morning of a big test in your mathematics class, you were involved in an automobile accident on your way to school. No one was injured, but there was some damage to your car.

Q20. Would the event of being involved in an accident on the way to school affect *your* performance on the test? In other words, would *you* consider these two events independent, or not?

Q21. Do you think that *everyone* would answer the previous question the same way?

Q22. Can you think of someone whose performance on the test would be affected by having been in an accident on the way to school?

Q23. Can you think of someone whose performance would *not* be affected?

Q24. Give an example of a pair of events that might for some people be independent, but for others not.

10.1.2 Should We Invite 240 People?

Mary Ann and Chris return to the question of how many people they should expect to attend if they invite 240. They have identified 6 possible outcomes, they know how many people would attend for each outcome, but they have learned that the 6 outcomes are not equally likely. They are sure that they cannot just average the 6 possible numbers of people attending, because they are not equally likely to occur. But they are not sure what to do. Once again, they consult Amy Smith. She suggests that, because the 6 possibilities have different probabilities, they could use a weighted average. That is, each of the 6 possible numbers of attendees could be weighted by its probability. Amy applied her weighted average idea like this:

$$\begin{aligned}\text{Average number attending} &= 114(0.12)+108(0.08)+152(0.30)+144(0.20)+171(0.18)+162(0.12)\\ &= 13.68+8.64+45.6+28.8+30.78+19.44\\ &= 146.94\end{aligned}$$

The weighted average that Amy Smith calculated is called the **expected value** of a **random variable**. The *random variable* in this case is the number of people who attend. This number is a variable, because it can have more than one value. It is a *random* variable, because its value depends on random events. In this case, the random events are the acceptance rate and the percentage of no shows. Finally, the *expected value* of the number of people who attend is 146.94. Notice that this expected value is not an integer.

Q25. Why is it acceptable to have a non-integer for the expected value of the number of people who attend?

Q26. If we rounded 146.94 to 147, based on the data in the preceding tables, could we actually observe 147 people attending the wedding based on Amy Smith's estimates

Q27. If 146.94 is the expected value of the number attending out of 240 people who are invited, what is the expected value of the *percentage* of invited guests who will attend?

It is important to remember that the expected value of a random variable is really a weighted average. Each of the possible values of the random variable is weighted by the probability that it occurs. Because probability can be thought of as a percentage, there is no need to divide the sum of the weighted values, because division is built into the probabilities.

10.1.3 Should We Invite More Than 240 People?

Chris and Mary Ann decide to explore what might happen if they invite more guests than their maximum. Amy Smith, their wedding planner, suggests that they consider the expected number attending for several different numbers of invitees. They decide to consider inviting 270, 300, and 330 guests. Table 10.1.3

shows the likely number attending and its probability for each of the six possible outcomes if 270 guests are invited.

Q28. What is the probability that the likely number of people attending is less that 175 if they invite 270 guests?

Q29. Complete a table similar to Table 10.1.4 if 300 or 330 guests are invited. Then use the likely number of attendees and their respective probabilities to find the expected value of the number of attendees for each of the three cases.

Acceptance rate	**% of no shows**	**Likely # attending out of 270***	**Probability**
One-half	10%	122	0.08
One-half	5%	129	0.12
Two-thirds	10%	162	0.20
Two-thirds	5%	171	0.30
Three-fourths	10%	183	0.12
Three-fourths	5%	193	0.18

* Fractional numbers have been rounded up to the next integer in each case.

Table 10.1.4: Likely number attending and probability if 270 are invited

Q30. If they decide to invite 300 guests, what is the probability that the likely number attending is less than 200? How does this probability change if they decide to invite 330?

Recall the goal that Mary Ann and Chris have: To maximize attendance at their wedding without exceeding 240 attendees. They have now considered inviting 240, 270, 300, and 330 guests.

Q31. Does the expected value of the number attending exceed 240 for any of these cases?

Q32. If the expected number attending is less than 240, does that mean that the actual number attending will not exceed 240? Explain.

Q33. Is there any case for which Chris and Mary Ann can be sure that the number attending will not exceed 240? Explain.

Q34. How many guests do you think that Mary Ann and Chris should invite? Explain.

Section 10.2: Investment in Automation

Boss Controls (BC) is an automotive supplier that manufactures integrated cup holders with temperature controls for car-makers throughout the world. Their new model is to be made available on one million new luxury cars worldwide. Initial estimates that car buyers will select this option are as high as 50% of the time or as low as 30% of the time. Because of a general decline in the global economy, BC marketing estimates there is a slightly higher chance that demand will be 30% rather than 50%. Management assigns a 0.55 probability that only 30% of the luxury car buyers will request their temperature controlled cup holders. Since the only other possibility they are considering is that 50% of luxury car buyers will request this option, a 0.45 probability is assigned to that possibility.

Q1. Why did they have to assign a 0.45 probability to the second possibility?

10.2.1 The Problem

Like most companies, BC wants to maximize its profit. BC's revenue comes from selling their cup holders to car companies at a fixed price of $60 apiece. The company can control manufacturing costs by how heavily it invests in automation. An investment of $13 million in high-speed equipment would lower the cost of producing each cup holder to $12. With a more modest investment of $8 million, the cost would be $27 per cup holder. Therefore, the net profit per unit after a high or low investment in automation will be $48 or $33, respectively. This per unit profit does not yet include the investment cost.

Q2. What decision must be made in this situation?

Q3. What is the **chance event** that BC faces? (i.e., an event whose likelihood must be predicted using probability and is outside the control of BC's management)?

10.2.2 Building a Decision Tree to Solve the BC Problem

In situations such as this, managers need to take a systematic approach to determine the best strategy for achieving their goal. We will build a **decision tree** similar to the probability tree in the last section to model the situation and show all the possible outcomes, their probabilities and their consequences.

Decision trees are made of *nodes* and *arcs*. The nodes on a decision tree represent decisions or random events. The arcs connect these nodes, showing paths that represent possible scenarios, depending on the outcomes of decisions and random events. To distinguish between the two types of nodes, decision and random chance, they are represented differently in a decision tree. Decision nodes are represented as rectangles and random chance nodes as circles, as in Figure 10.2.1.

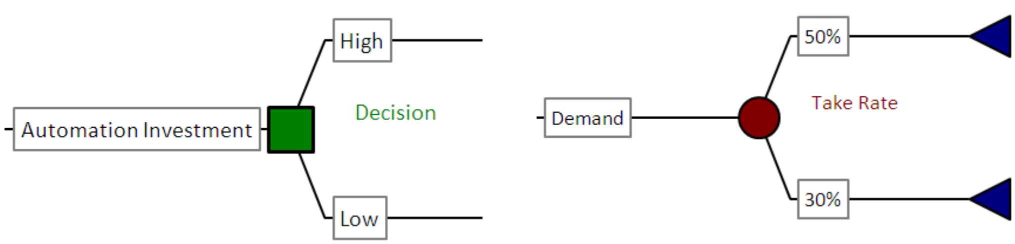

Figure 10.2.1: Example of decision node and chance node

In the BC scenario, whether there will be a high or low level of investment is a *decision* that management must make. However, whether the demand for their product will be 50% or 30% is strictly a random event, and BC's management has no control over it. The two elements, decision and random chance, can be combined into a decision tree that models the entire Boss Control scenario as shown in Figure 10.2.2.

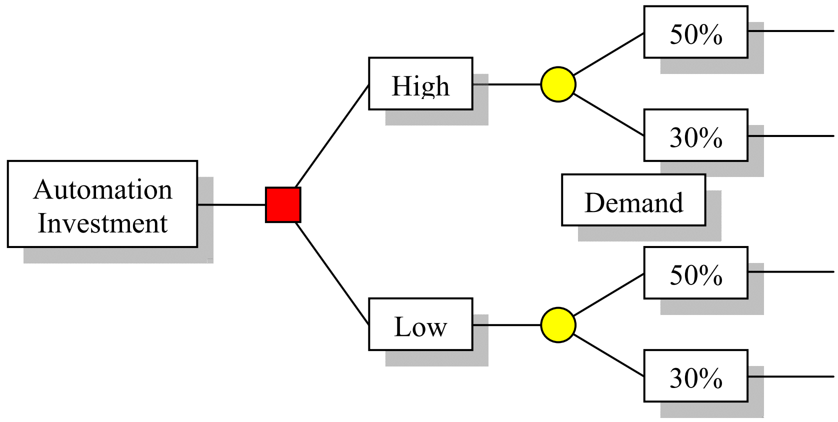

Figure 10.2.2: Decision tree for BC automation investment

This decision tree shows all the possible outcomes for every combination of decisions and random events. Now we must add the appropriate data to determine the expected profit of each path through the decision tree.

Recall that the low investment in automation was going to cost BC $8 million and the high investment was going to cost $13 million. This information can be added to the decision tree by placing the numbers along the appropriate branches, below the arcs. (The space above the arcs is reserved for another quantity, as you will see shortly). In this problem, the units on all such numbers will be millions of dollars. The numbers below are negative because they represent expenses BC must pay, not revenue earned.

Recall also that the unit profit after a low investment in automation is $33 whereas the unit profit after a high investment is $48. The "30%" and "50%" after the chance nodes represent the proportion of car buyers who select the luxury cup holder option. This is referred to as the *take rate*. In BC's case, the estimated low take rate is 30% and the estimated high take rate is 50%.

Q4. What is the estimated probability that the take rate will be 30%?

Q5. What is the estimated probability that the take rate will be 50%?

Q6. What is the base of these take rates? In other words, what is the total number of potential sales?

The net revenue for each case can be calculated by multiplying the projected number of buyers times the net profit per unit. The projected net revenue with a 50% take rate and *high* investment is

Net Revenue = 1,000,000*.5*$48 = $24 million

Q7. What is the projected net revenue with a 30% take rate and *high* investment?

Q8. What is the projected net revenue with a 50% take rate and *low* investment?

Q9. What is the projected net revenue with a 30% take rate and *low* investment?

In Figure 10.2.3, the decision tree has been updated to include the information above. As before, the numbers beneath the arcs are in millions of dollars. They represent cost or net revenue. The numbers assigned to arcs coming from a random event show the probability of that path.

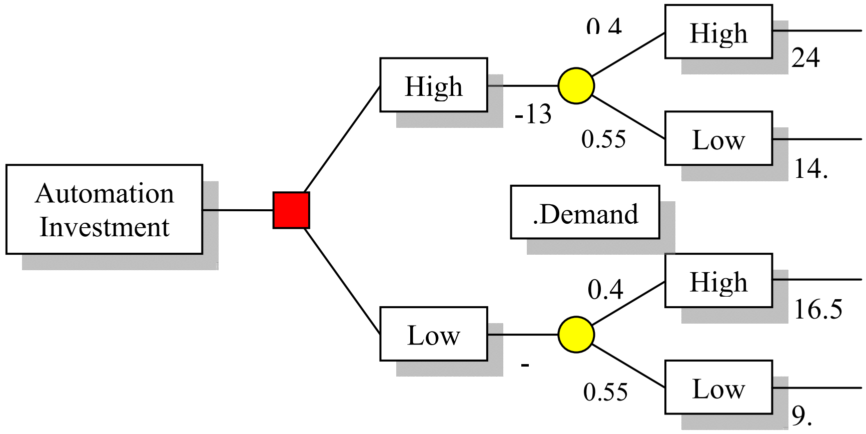

Figure 10.2.3: Decision tree showing dollar values and probabilities

An end node, represented by a triangle, is placed at the end of each path through the decision tree. Next to the triangle, we record the net profit for that path. The net profit is determined by subtracting the investment cost from the net revenue. For example, with a High Investment of $13 million dollars and a 50% take rate, the net profit is:

$24 million – $13 million = $11 million.

Figure 10.2.4 has been updated to show each of the four possible net profit values to the right of the end nodes.

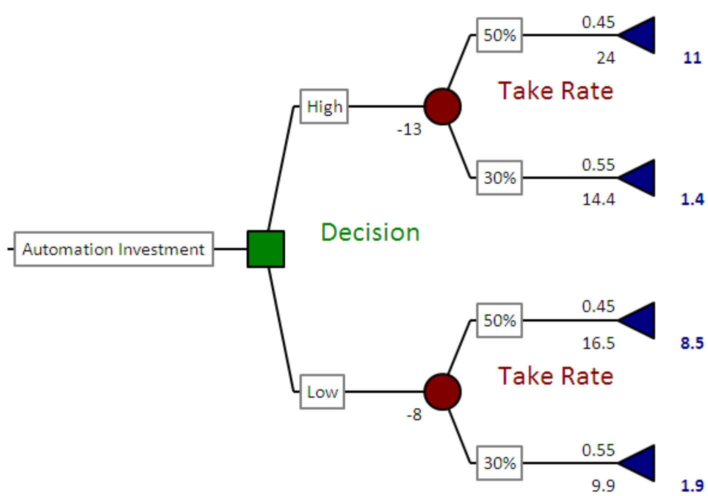

Figure 10.2.4: Decision tree showing probabilities and expected values at end nodes

The next step is to determine the expected value of the net profit for each decision. We simply use the basic formula for expected value. The expected value of the net profit for a high investment is calculated as follows:

If we make the high investment decision, there is a 0.45 probability that the net profit will be \$11 million. There is a 0.55 probability that the net profit will be only \$1.4 million. Thus, the expected value of the net profit for the High Investment decision is:

Expected Net Profit = (0.45)(\$11 million) + (0.55)(\$1.4 million) = \$5.72 million

Q10. What is the expected value of the net profit for a low investment?

Q11. Is it clear from your answers whether BC should make a low or high investment in automation? Why or why not?

Q12. Which of the two options, high or low investment in automation, has the larger expected value for net profit? How much larger than the other option is it?

Q13. Suppose the vendor for the High Speed automation has just announced a \$500,000 price increase from \$13 million to \$13.5 million. Would this affect the preferred decision? What if the increase was a million?

10.2.3 Boss Controls Sensitivity Analysis

The management at Boss Controls now wonders how sensitive the solution to their automation problem is to their probability estimates of the take-rate. Recall that they have estimated the probability of a 50% take-rate to be 0.45 and the probability of a 30% rate to be 1–0.45=0.55. But, they wonder, "What if our estimate of the probability of a 50% take-rate is too high? Could that change which alternative has the larger expected value of profit? If so, at what point does the larger expected value change from high investment to low investment?" In other words, they want to learn how low the probability of a 50% take-rate could be and still have the high investment in automation as the alternative producing the larger expected value of the profit.

Q14. Notice that the BC managers are not wondering what would happen if their estimate of the probability of a 50% take-rate is too low. Why not?

The decision to make either a high or low investment in automation rests on the expected value of the profit for each of the alternatives. Recall that those expected values are given by:

Expected value of profit for high investment = \$11 million(0.45)+\$1.4 million(1–0.45)
Expected value of profit for low investment = \$8.5 million(0.45)+\$1.9 million(1–0.45)

Now, for each alternative, we can define a function to calculate the expected value of the profit based on varying probability estimates.

Let: x = the probability of a 50% take-rate,
y_1 = the expected value of profit for high investment, and
y_2 = the expected value of profit for low investment

Then, $1-x$ = the probability of a 30% take-rate,
$y_1 = 11(x)+1.4(1-x)$, and
$y_2 = 8.5(x)+1.9(1-x)$.

Notice that in both cases we have dropped the "\$" and the "million." We can do this, because we are using the same unit of measure, millions of dollars, in each case.

Figure 10.2.5 shows the set-up and calculator graphs of these two functions on the same coordinate axes. Set-up and graph this system on a graphing calculator.

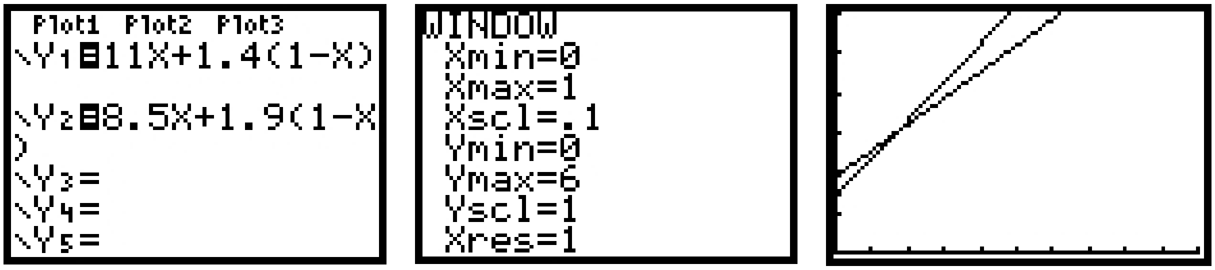

Figure 10.2.5: Expected value functions, window, and graphs for sensitivity analysis.

Q15. The viewing window for the graphs has been set up so that x varies between 0 and 1, only. Why does that make sense for this problem?

Q16. What do the y-values of the functions represent? What are their units of measure?

Use the TRACE feature of your calculator to trace on the graph of y_1 until the value of x is 0.45 when rounded to two decimal places.

Q17. When $x \approx 0.45$, is y_1 above or below y_2? What does that mean in the context of the problem?

Q18. Use the calculator to find the expected value of profit for high investment if the probability of a 50% take-rate is 0.4? What is the expected value of profit for low investment for this probability of a 50% take rate?

Q19. What are the slopes of y_1 and y_2, the expected value of profit lines? What do these slopes mean in the context of the problem?

Q20. Notice that the graphs of the two functions intersect. What is the significance of the point of intersection?

Q21. Use the calculator to find the x-value of the point of intersection, rounded to two decimal places. What does this x-value tell you?

Q22. For what probabilities of a 50% take-rate does the high investment in automation produce the higher expected value of profit? For what probabilities does the low investment produce the higher expected value?

Section 10.3: Green Tree Energy—Location a New Plant

Green Tree Power, Inc (GTP) is planning to expand their energy company by building a new power plant in a developing country. After much consideration, they have narrowed their possibilities to the countries of Cassedonia and Kisanthia. Each country can provide the required land and utilities for GTP to build and run their new power plant. In turn, the selected country will gain the benefits of the new energy technology that GTP can provide. Choosing the ideal location relies on several key pieces of information.

In order to build the new power plant in Cassedonia, GTP estimates that the investment cost will be $50 million. However, there is significant uncertainty with regard to increased demand for power. As a result, the predicted total net revenue over the next five years is uncertain. Experts project that revenues could be as low as $80 million or as high as $110 million. The specific probabilistic forecast is that five-year total net revenues will be $80, $90, or $110 million with a 30%, 40%, and 30% chance, respectively. The political structure in Cassedonia is in transition. There are multiple political parties fighting for control of the country. These political parties have very different social and economic plans. Thus, the leadership of GTP believes there is a 20% chance that a Cassedonian government will take over the new power plant. If so, the government will simply repay the original investment cost of $50 million with no interest. In that case, GTP would have no net revenue gain from its investment.

If GTP builds the new power plant in Kisanthia, the investment costs are still $50 million. The population of Kisanthia is slightly lower than in Cassedonia but demand is still uncertain. GTP estimates the total net-revenue for a five-year period in Kisanthia, after the operation costs, will be $66, $80, or $90 million with a 30%, 40%, and 30% chance, respectively. The country of Kisanthia is a long established stable democracy that is committed to encouraging foreign investments. While the total forecasted revenue in Kisanthia is significantly lower than in Cassedonia, it has a major advantage. There is little chance that the Kisanthian government will take over the new power plant.

Where should GTP build their new power plant?

10.3.1 Developing the Decision Tree

Joe Riden, a senior risk analyst from GTP, has been charged with making this decision. In order to weigh the options carefully and objectively, Joe will create a decision tree to analyze the situation and determine the best choice for GTP.

In our situation, we have one decision to make with two possible options—GTP's new power plant can be built in Cassedonia or Kisanthia. Joe decides to develop the decision tree in stages. First he places the two alternative decisions at the first decision node. The uncertainty with regard to Kisanthia involves just the net revenue. He therefore, adds the random event, net revenue, to the Kisanthian branch of the tree. He includes all of the critical information at the appropriate places. He places a -50 on the decision branch to represent the investment cost. The net revenue random event has three branches, high, medium, and low. For each branch, Joe inputs the probability and the net revenue values. For the High revenue branch the probability is 0.3 and its projected revenue for Kisanthia is $90 million. He also inputs the corresponding numbers for Medium (0.4, $80 million) and Low (0.3, $66 million). This first stage of Joe's tree construction is presented in Figure 10.3.1.

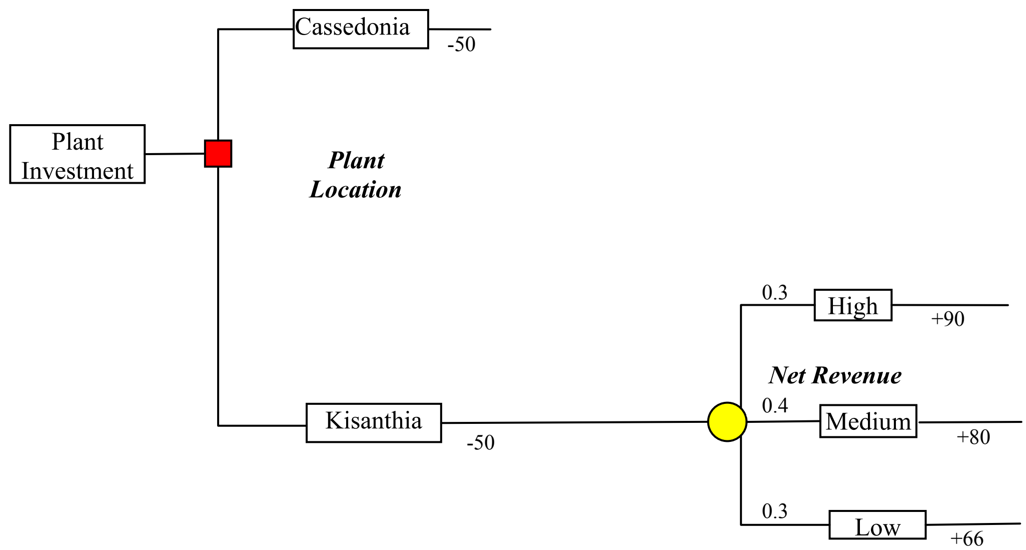

Figure 10.3.1: GTP—Kisanthian Alternative

Next Joe adds the uncertainties related to building a plant in Cassedonia. Recall there is a 20% chance that the Cassedonian government will take over the power plant after it is completed. We therefore need a chance node off of Cassedonia. The random event, a government takeover, has two branches: yes and no. If the government seizes control of the power plant, they will repay the $50 million dollar investment and GTP will no longer be involved with the plant. This return of $50 million is included along the Yes branch for a government takeover.

However, there is an 80% chance that the government will not take over the new power plant and GTP can move on to producing power, and thus revenue. Down this branch, there is another random event, uncertain net revenue. Joe attaches another random node with three branches for the net revenue. Again, each branch has a probability and dollar amount for net revenue: High (0.4, $110 million) Medium (0.4, $90 million) and Low (0.3, $80 million). Joe added the probability of a takeover and the net revenue uncertainty as presented in Figure 10.3.2.

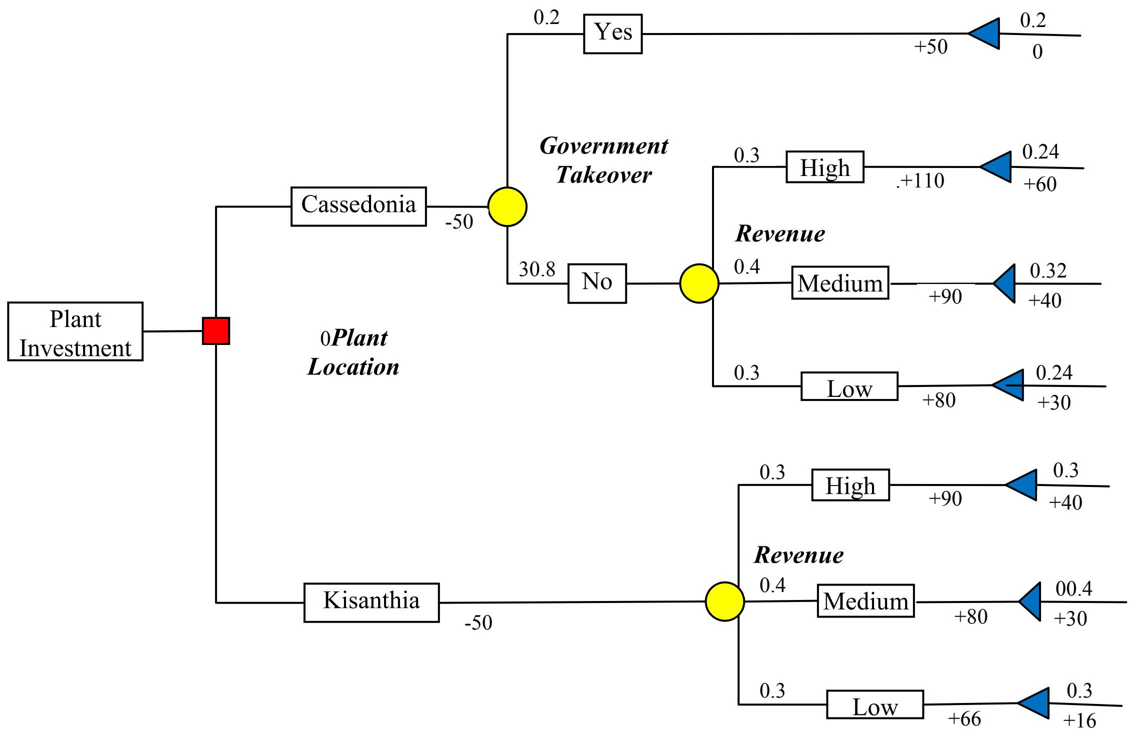

Figure 10.3.2: GTP – Cassedonian Alternative Added

In order to make the decision, we need to perform some calculations. At each end node, we want to note two important values: the probability of following that path from start to finish and the total profit for GTP if they follow that path. For example, if GTP were to build in Kisanthia and net revenues turn out to be high, the net profit would be $40 million: $90 million in net revenue minus $50 million investment. The probability of that happening is just 0.3. He then adds the probabilities and net profits for the other two branches.

For Cassedonia, if the government takes over the plant, the net profit is zero. This has a probability of 0.2 of occurring. If the government does not takeover the plant and the net revenue is high, the net profit is $60 million ($110-$50). The likelihood of this happening is the product of two probabilities. Mr. Riden multiplies the probability of no government takeover, 0.8, by the probability of high net revenue, 0.3. The product is 0.24. He repeats this process for the other two possible branches for net revenue for Cassedonia. Figure 10.3.3 contains all of the final dollar values and probabilities placed alongside the end nodes. The probability is placed above the net profit value.

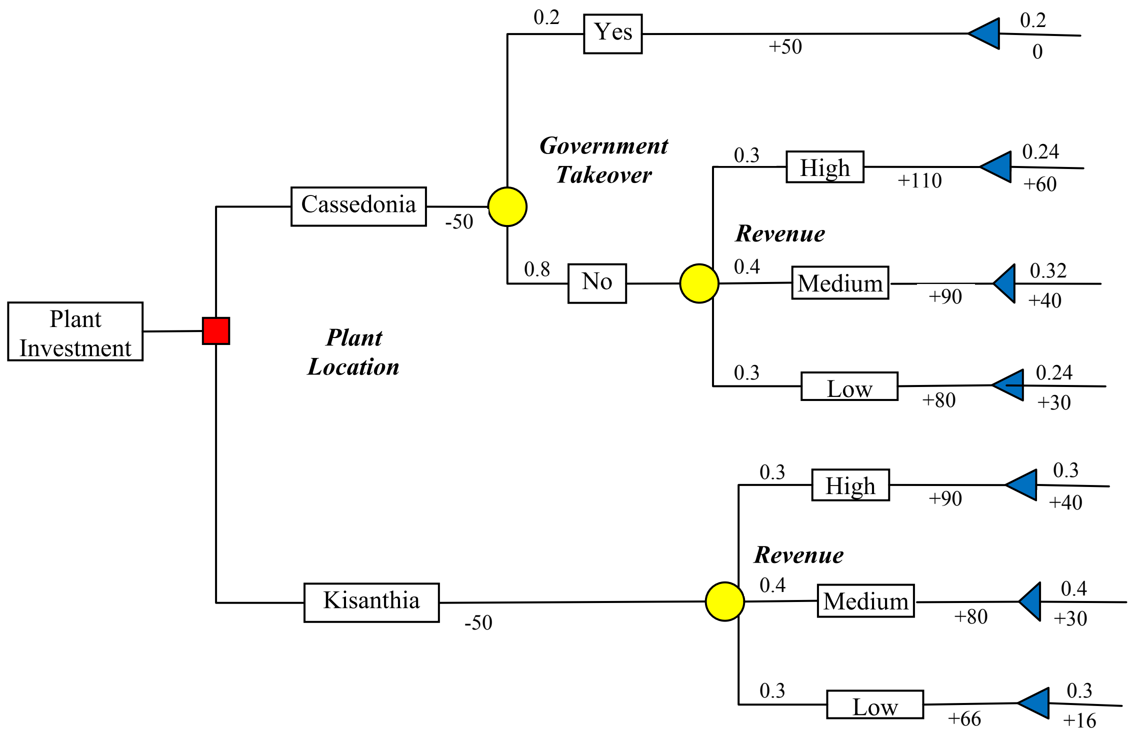

Figure 10.3.3: GTP Decision Tree

10.3.2 Computing the Expected Value

The final step in making the decision is to determine the expected value of the net profit for each alternative decision. We need the expected value of the net profit for building the power plant in Cassedonia and the expected value of the net profit for building in Kisanthia. The country with the larger expected value will be the best option for GTP.

The expected value for net profit of building the plant in Kisanthia is determined by taking the weighted sum of the different possible net profit values. There is a 30% chance of making $40 million, a 40% chance of making $30 million, and a 30% chance of making $16 million. So, the expected value of net profit is:

(0.3)($40 million) + (0.4)($30 million) + (0.3)($16 million) = $28.8 million

Consider the option of building in Cassedonia. There is a 20% chance of making $0, a 24% chance of making $60 million, a 32% chance of making $40 million, and a 24% chance of making $30 million. Thus, the expected value of net profit for building the plant in Cassedonia is:

(0.2)(0) + (0.24)($60 million) + (0.32)($40 million) + (0.24)($30 million) = $34.4 million

Q1. Based on the expected values, which country is preferred for the investment?

Q2. What is the minimum net profit for Cassedonia? How likely is that to occur?

Q3. What is the minimum net profit for Kisanthia? How likely is that to occur?

Q4. How might the issue of risk affect your preferred decision?

10.3.3 GTP, Inc. Considers Insurance

Freud's of London understands the psychology of risk. It offers specialty risk insurance for large projects. Freud's is prepared to offer GTP insurance against a possible government takeover in Cassedonia. They are prepared to charge GTP a $3.5 million upfront premium to insure against a government takeover. If the government takes over GTP plant, Freud's will pay GTP $10 million dollars. We will analyze this insurance policy both from GTP's perspective and that of Freud's. The GTP part of the tree is revised and presented in Figure 10.3.4. a negative $3.5 million dollars for the insurance premium is added to the cost of the Cassedonia branch. That total cost is now $53.5 million. As a result each end value for NO government takeover decreases by $3.5 million. However, the end node for the YES government takeover now has a net profit of $6.5 million dollars. They receive $50 million back from government plus a $10 million payment from the insurance company for total revenue of $60 million against costs of $53.5 million. This yields a net revenue of $6.5 million.

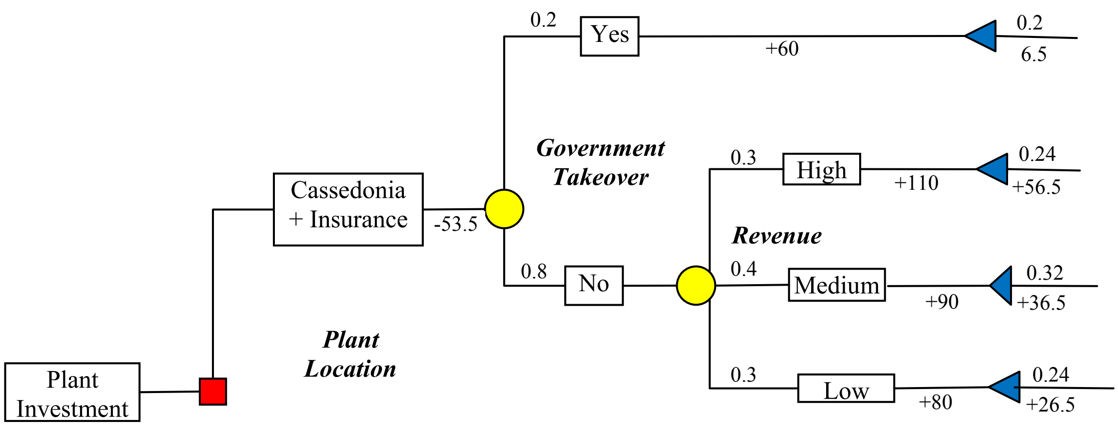

Figure 10.3.4: GTP Cassedonia with Insurance

The new expected value of net revenue for the Caledonian option can be calculated:

(0.2)($6.5 million)+(0.24)($56.5 million)+(0.32)($36.5 million)+(0.24)($26.5 million) = $32.9 million

The expected net profit has declined by $1.5 million. However, GTP is assured now of making at least $6.5 million from its $50 million investment.

Q5. Would you recommend buying the insurance?

The above analysis focused on GPT's perspective. Let's look at the decision from the insurer's perspective of Freuds of London. If they offer insurance, they face only one uncertainty, a government takeover. They are uninterested in the uncertain event, revenue. Their decision tree is presented in Figure 10.3.5. If they sell no insurance, they gamble no money and they make no profit. If there is a government takeover, Freud's must pay GPT $10 million. Their net loss would be $6.5. With no takeover, their profit is the premium they charged.

Q6. How much risk does Freud's face?

Q7. What is the expected value of the profit Freud's will earn?

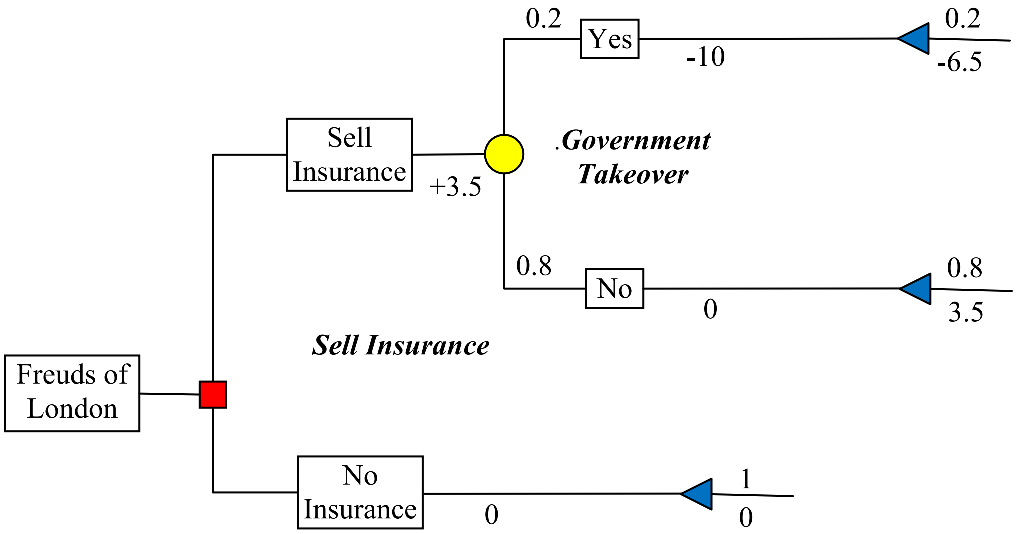

Figure 10.3.5: Freud's of London Insurance Decision

The early decision analysts recognized that expected value alone did not represent how many individuals and companies deal with risk. Many of us are "risk averse." This means we would be willing to accept a reduced expected value in exchange for more certainty. This is the reason people buy insurance. They are willing to pay money to avoid risk. The cost of the insurance is always more than the expected value of the loss. We pay hundreds of dollars to insurance against a relatively unlikely catastrophic risk to our homes. We buy family medical insurance for thousands of dollars to cover the cost of a major surgery and a long hospital stay that could cost more than a hundred thousand dollars. As individuals living one life and facing one situation, we cannot rely on long range expected value. However, insurance companies can tolerate these types of risks and live by the expected value. They can pool the risk across thousands of customers each year. Their financial performance will approximate the expected value.

Decision analysts developed a concept called utility theory to quantify this concept of risk aversion. The mathematics of utility theory is beyond the scope of the course. Instead, we will present tradeoffs between reduced expected value and more certainty and let you judge your preference. In the next example of automobile collision insurance, we will challenge you to revisit your own attitude towards risk when making smaller insurance decisions

Section 10.4: Purchasing Collision Insurance

Jee Min is a high school junior at Cassidy High School in Thomasville, Michigan. He has been an excellent driver for one year. With the help of his parents, he has just purchased a 2005 Chevrolet Cavalier. Jee drives his car to school and to his part-time job on the weekends and after school. He is considering purchasing collision insurance for this car. It has a *Kelly Blue Book* value of $6600.

After gathering quotes from insurance companies through the internet, Jee has learned that the lowest six-month premium for collision insurance with a $500 deductible is $1700. He is not sure that he can afford that much, so he decides to investigate the cost of collision insurance with a $1000 deductible. The six-month premium for collision insurance with a $1000 deductible is $1500. The deductible represents the maximum amount of loss the owner incurs in the case of an accident. With a $500 deductible, the owner must absorb the cost of the first $500 of damages. For example, if the damages were $250, he bears the whole cost. If the damages were $6600, he absorbs the $500 and the insurance company pays him $6100. Jee Min is also considering the possibility of not carrying any collision insurance. In order to make the best decision, he must consider all of the consequences of each possibility.

To examine his options, Jee created a decision tree showing the three collision insurance possibilities. Each branch of the tree is labeled. Let's examine his decision tree. The tree begins with a decision node, because Jee must decide whether to purchase collision insurance with a $500 deductible, a $1000 deductible, or no collision insurance at all. The next node on each of the three branches is a probability event: a collision in the next 6 months. He chose a 6 month time period for this event, so that it matches the six-month period covered by the premium quotes that he obtained. Next, Jee Min needed an estimate of the probability that he would be involved in a collision in the next six months.

After some Internet research, Jee Min learned that according to the National Highway Safety Administration, the probability that a male teenage driver in the U.S. will have an accident in any six-month period is 30%. He decided to use this probability estimate. Then he attached branches to the probability nodes to account for the possibilities that he will have an accident or not during a given six-month period.

Q1. How do you think the National Highway Safety Administration determined the probability a male teenage driver will have an accident in a six-month period?

Q2. Why was 70% assigned to the branches representing the event that Jee Min will not have an accident in the next 6 months?

Let's explore the structure of the tree (See Figure 10.4.1).

Q3. There are three branches that leave the main rectangular decision node. What do these three branches represent?

Q4. Each of these three branches leads into a random circle node. What does that random node represent? Why are there two branches coming out of this node?

If Jee Min does have an accident, various damage amounts are possible. He realized that even if he rounded damage amounts to the nearest dollar, there would still be 6,600 possible damage amounts. Therefore, he decided to list these amounts in ranges. He also investigated the probabilities that the damage amount of an accident falls within that range. Jee Min also collected this probability data online from the National Highway Safety Administration. He organized this information in Table 10.4.1.

Damage Amount Ranges	Probability
Less than or equal to $500	45%
Greater than $500 but less than or equal to $1000	15%
Greater than $1000 but less than or equal to one-half the value of the car	25%
Greater than one-half the value of the car but less than or equal to $6,600, the total value of the car	15%

Table 10.4.1: Damage Estimate Probabilities

Based on this data, Jee Min added this information to the decision tree. He added another probability node to each branch of the tree that represents having or not having an accident in the next 6 months. Then he added a branch for each of the possibilities in Table 10.4.1 to each new node. Finally, he labeled each new branch with the probability of incurring that amount of damage. Note the number of sequences of branches in Jee Min's tree. The probability of that sequence of random events is listed at the end of each sequence of branches.

Q5. Each branch labeled YES contains another random node? What does this represent? Why are there four branches leaving this second node?

Let's explore the probabilities on the tree. There is a probability assigned to the end node of each sequence of branches. For example, the top end node has a 0.135 assigned to it.

Q6. How was this end probability determined?

Q7. Which end nodes have the highest probabilities and why?

Q8. The probabilities for the top five branches in the picture sum to 1. Why is this the case?

Let's discuss the dollar amounts of the tree and the end node values.

Jee Min realized that for each complete decision branch of his tree, he must assign a cost to each branch. However, in order to do these calculations, he realizes that he needs *individual* damage amounts and not *ranges* of damage amounts. Jee Min decides to use a single number within each range to represent the damage amount for that branch. He listed those in the following table.

Damage Amount Ranges	Representative Amount
Less than or equal to $500	$250
Greater than $500 but less than or equal to $1000	$750
Greater than $1000 but less than or equal to one-half the value of the car	$2150
Greater than one-half the value of the car but less than or equal to the total value of the car	$6600

Table 10.4.2: Damage Estimate -Representative Amounts

For the three lowest damage ranges, Jee Min decided to use the midpoint of the range: $250 for accidents having damage less than or equal to $500, $750 for accidents having damage greater than $500 and less than or equal to $1000, and $2150 for accidents having damage greater than $1000 and less than or equal to one-half the $6600 value of his car. For the last range, damage greater than one-half value and less than or equal to the full value of his car, he learned that insurance companies almost always "total" the car when the damage falls within this range. Therefore, he decided to use $6,600 for accidents in that range.

All four of these dollar amounts are entered into the appropriate branches of the tree for the no collision insurance decision. Now let's look at the section of the tree with the decision, $1000 deductible, and the random event the damage. The dollar amounts for these four branches do not match the values in table.

Q9. Which two branches match the table and which two do not? Why?

Q10. For the section of the tree with a $500 deductible, there are three branches with a $500 value. Why is this the case?

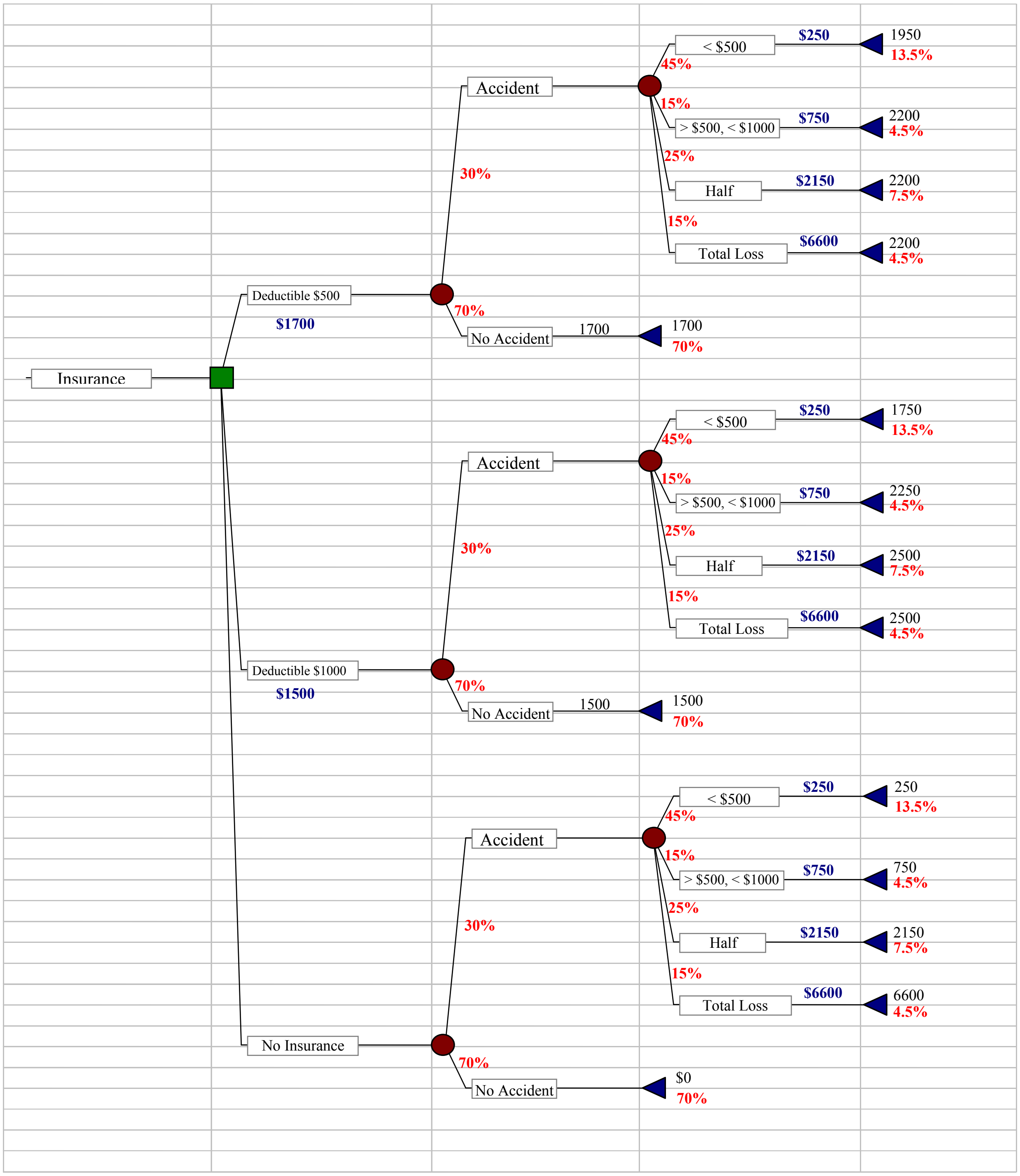

Figure 10.4.1: Collision Insurance Decision Tree

Consider only the portion of the entire decision tree that represents the decision to purchase $500 deductible collision insurance. Recall that the premium for this deductible amount is $1700. For the top-most branch, the cost is $250. Thus, the end node value is the sum of these two costs, $1950. The second branch, has a $500 cost. The end node value is therefore $2,200.

Q11. Why are the end node values for the third and fourth branch the same $2,200 as for the second branch?

In a similar way, Jee Min adds the cost of the premium with his liability for damages to determine the end node value for each sequence of branches.

Q12. What is the expected cost to Jee Min of the decision to purchase $500 deductible collision insurance?

Q13. Examine each of the remaining decision options and the total cost. In a similar way, determine the expected cost to Jee Min of making that decision. Enter each of the expected costs in the table below.

Decision	**Expected Cost**
$500 Deductible	
$1000 Deductible	
No Insurance	

Table 10.4.3: Expected Costs

Q14. Based on this analysis, which option has the smallest expected cost?

Q15. Should Jee Min base his decision only on this analysis? Explain why or why not.

The basis for the insurance industry recognizes the concern people have with incurring huge costs associated with relatively infrequent events. Every reduction in premium fees can be directly subtracted from the total expected value to determine the impact.

Q16. Given your own attitude towards risk, would you be willing to pay more than the expected value calculated for the no insurance options to reduce your risk? If so, how much would you be willing to pay to have a policy with a $1,000 deductible?

10.4.1 A Revised Estimate of the Probability of Having an Accident

Over the past six years, Michigan and other states in the U.S. have instituted graduated driver training programs for new teenage drivers. In fact, Jee Min participated in such a program. The establishment of these programs has resulted in safer teenage drivers.

What if the National Highway Safety Administration has now determined the probability that a teenage driver in the U.S. will have an accident in any six-month period is 22%.

Q17. Will this new estimate of the probability of having an accident affect the expected cost for each decision?

Q18. Using this new estimate, what is the probability of *not* having an accident in the six-month period?

The insurance companies are slowly responding to this reduced rate of accidents. They are considering significant reductions in the premiums charged.

Q19. If the premiums were reduced by 30%, what would be the expected values for each of the insurance policies?

Q20. Recalculate the expected cost for no insurance option. Should Jee Min change his decision? Explain.

Section 10.5: Chapter 10 (Decision Trees) Homework Questions

1. The probability of selling a dress in a store is 25 percent each week.
   a) Construct a probability tree to determine all of the possible of outcomes over a three-week period.
   b) What is the probability that the dress will not be sold at the end of the third week?

2. A contestant on a TV show must pass four stages to win a big prize. The probabilities of winning in stage 1, 2, 3, and 4 are 0.8, 0.6, 0.4, and 0.3 respectively. The contestant wants to know the probability of winning the big prize.
   a) Construct a probability tree to determine the possible outcomes of the game.
   b) What is the probability that he wins the prize?
   c) What is the probability that he makes to stage 4 but does not win the prize?

3. A TV cable company has a technical support department to solve customers' problems by phone. In this department the staff is categorized in four levels based on their ability to solve customer problems. The company first assigns a problem to level one; if they cannot solve it, someone at level two is assigned. This process is repeated until it finally reaches the most experienced staff for one last attempt at solving the problem. The probability that a staff person is able to solve the problem at each level is 0.50, 0.75, 0.85, and 0.95 consecutively.
   a) List all of the possible outcomes.
   b) Construct a probability tree to determine the probability of each outcome.
   c) What is the probability that a customer's problem is unsolved?
   d) What is the probability that the problem is solved by someone at level 3?
   e) Why is this probability less than the probability that a level 1 individual solves the problem?

4. A manager at Wayne State football games must decide ten days in advance which product to order for the stadium vendors to sell. Each product will have the university logo. The three options are sun visors, umbrellas, and ponchos. He will stock only one of the items. Sales and the resultant profit will depend upon the weather on the day of the game. The long-range weather forecast is 35% chance of rain, 25% chance of overcast skies, and 40% chance of sunshine. The manager estimates that the following profits will result from each decision and each weather condition.

	Weather Condition		
Decision	*Rain(.35)*	*Overcast(.25)*	*Sunshine(.40)*
Sunglass	-600	-300	1600
Umbrella	2100	0	-800
Poncho	1800	500	-600

   a) What is the best decision for each weather condition?
   b) Draw the associated decision tree needed to make the best decision.
   c) What decision should be made if he desires to maximize the expected value?

5. The owner of a restaurant is considering two ways to expand operations: open a drive-thru window or serve breakfast. There are increased annual costs which each option and a one-time cost associated with the drive-thru. Labor and marketing costs are annual costs that the restaurant has to pay each year that include hiring new staff and placing more ads in local media. Redesigning the restaurant is a one-time cost that is paid at the beginning and does not repeat each year. The detail is provided in the following table.

Decision	Costs		
	Annual		One Time
	Labor	Marketing	Restaurant Redesign
Drive-thru window	28,000	10,000	50,000
Breakfast	38,500	5,000	-

The forecasted increase in income resulting from these proposed expansions depends on whether a competitor opens a restaurant down the street or not. Based on the restaurants evaluation, the manager is sure that the competitor won't open a new restaurant with 60 percent of certainty. Based on the competitor action, the restaurant's profit will be different for each decision. Following table provides estimation in increase of income based on the competitor's action.

	Competitor	
Decision	Open(0.4)	Not Open (0.6)
Drive-thru window	110,000	130,000
Breakfast	80,000	120,000

The owner of the restaurant is focused just on next year. He therefore decided to consider the one-time cost for the redesign the same as all of the labor and marketing costs that are ongoing.

a) Calculate the profit of each decision when considering the competitor's action.
b) What is the best alternative if no competitor opens nearby? What is the best alternative if a competitor opens nearby?
c) Draw the associated decision tree.
d) What decision should the company follow?
e) Let *p* represent this probability that the competitor will open a restaurant down the street. Write an equation to calculate the expected value for each decision as a function of *p*.
f) For what value of *p* are these two expected values equal?
g) Graph the equations of the expected values to determine their intersection point. What does this intersection point represent?
h) Recall that the owner treated the design change and marketing cost the same as operating costs. Would the decision change if he considered only 50% of these costs this year (design and marketing)?

6. A company is about to launch its new fast food for sale in supermarkets throughout Arkansas. The research department is convinced that a special type of chicken wings will be a great success. The marketing department wants to launch an intensive advertising campaign. The advertising campaign will cost $1,000,000 and if successful will produce $4,800,000 profit. If the campaign is unsuccessful (25% chance), the profit is estimated at only $1,800,000. If no advertising is used, the revenue is estimated at $3,500,000 with probability 0.6 if customers are receptive and $1,500,000 with probability 0.4 if they are not.

a) Draw the associated decision tree.
b) What course of action should the company follow in launching the new product if they want to maximize the expected value?
c) Write an equation to calculate the expected value for each decision as a function of the probability that the major advertising campaign will be effective?
d) Graph the equations of the expected values to determine their intersection point. What does this intersection point represent?

7. A discount clothing store uses an interesting strategy to attract customers to return each week to shop. They tell the customers that every 7 days they reduce by 25% the original price of any unsold dress. On each dress there is a label of its original price and the date it was hung on the rack. Thus customers know that a $40 dress placed out on Nov. 7th will be priced only $30 on Nov. 14th if it is not sold before then. It will be reduced by another $10 on Nov. 21st if it is still unsold. After three weeks, any unsold dress is sent to a local charity. Each week, there is a .60 probability that the dress will be sold.

   a) Nancy Drew saw a dress she really liked and knows she can get the almost identical dress for $50 online. The current store price is $40. Construct a decision tree to determine whether or not she should buy the dress now or gamble and wait a week and buy it next week if it remains unsold. (If when she comes back next week, Nancy finds the dress has been sold, she will buy it online.)
   b) Just before finalizing her decision, she found another place online that sells the same dress for $45. Why might a lower price online affect her purchase decision in this store? Should she buy the dress now or gamble and buy it in the second week if available?
   c) She just saw a more expensive dress for sale at $80. These more expensive dresses have only a 40% chance of being sold each week and again they tell the customers that every 7 days they reduce by 25% the original price of any unsold dress. If she would be able to buy a similar dress with $90 online, construct a full tree for 3 weeks.

8. A contestant on a TV show has to decide whether to stop or try to answer another question. The contestant is first asked a question about US Geography. If the contestant answers correctly, she earns $700. Historically three out of four contestants answer the first question correctly. If answered incorrectly, the game is over. If answered correctly, the contestant can leave with $700 or go on and answer a question about US presidents. If answered correctly, the contestant wins an additional $1000. If the answer is incorrect, the contestant loses all previous earnings and is sent home. Historically, two out of three contestants answer this question correctly. The third question is about rock 'n' roll music. This question is worth $1500, and the same rule applies. The chance of answering this question correctly is 50-50.
   a) Draw a decision tree that can be used to determine how to maximize a contestant's expected earnings. What is the best decision and what are the expected earnings in this case?
   b) Some contestants may feel more or less knowledgeable about the third question category. Let p represent the probability that a contestant will answer the third question correctly. Write an equation to calculate the expected value for attempting to answer the third question in terms of p.
   c) Based on the previous question, what is the cutoff value of p such that a contestant should attempt the third question?
   d) The TV show is considering changing the reward for answering the 3rd question correctly. Let m represent the amount of money a contestant will earn for answering the third question correctly. Write an equation to calculate the expected value for the last decision as a function of m.
   e) Graph the equations of the expected values to determine the intersection point for the last stage. What does this intersection point represent?

9. SSS Company, a software company, is considering submission of a bid for a state government contract to install their software on 30,000 computers. The government would use their software to oversee the management of tens of thousands of large and small contracts the government signs every year. There is only one other potential bidder for this contract, Complexo Computers, Inc. Complexo has a long record and reputation with this kind of contract. As a result of its lesser experience, to win the bid SSS's bid must be at least $5 less per computer installation than Complexo's. Complexo Computers is certain to bid and is generally more expensive than SSS. SSS management believes that it is equally likely that Complexo will bid $100, $90, or $80 per computer installation.

a) What are the possible bids that SSS should consider?

SSS's bidding decision is complicated by the fact that it is currently working on a new process to install software remotely through the internet. If this process works as hoped, then it may substantially lower the cost of installations. However, there is some chance that the new process will actually be more expensive than the current installation process. Unfortunately, SSS will not be able to determine the cost of the new process without actually using it to install the software. The higher SSS bids the more money it makes if it wins the contract. However, the higher the bid, the less likely it is to win the contract. If SSS decides to bid, it will cost $20,000 to prepare all of the relevant documents required to submit the bid. SSS will incur this expense regardless of whether it wins or loses the bidding competition. With the proposed new installation process, there is a 0.25 probability that the cost will be $50 per computer and a 0.50 probability that the cost will be $75 per computer. Unfortunately, there is also a 0.25 probability that the cost will be $85 per computer.

b) Construct a decision tree to model this situation.
c) Based on your decision tree, do you recommend SSS Company to submit a bid, and if so, what should they bid per installation?
d) Under the optimal policy, what is the probability they will win the contract?
e) What is the overall expected value if they bid on the contract?
f) If they win the contract, what is their expected value of profit?

10. A group of high school students has decided to start a summer business. The work that they are thinking about is designing and coloring T-shirts and selling them to clothing stores in their community. For mass production of colored T-shirts, they need special equipment which they can buy or rent. After negotiating with a company about equipment, they figure out that they have three options to start their business:

- They can buy all the equipment and do the design and printing themselves. In this case they have to pay for equipment but they can recover part of the money at the end of summer by reselling the equipment. The cost of buying equipment is $8,100, and they can resell it at 50% of the original price. The cost of printing will be $1 per T-shirt.
- The second option is renting the equipment and returning it at the end of summer. The renting cost is $1,500 for the whole summer and a variable cost of $1.50 per print.
- The third option is outsourcing the printing. In this case they do the designs themselves but send them to a company for printing. The company charges them $2 per T-shirt. Note that in option one each T shirt costs them 1 dollar.

The fact that the market demand for colored T-shirts is not certain makes the decision making difficult. After doing some market evaluation, they summarized their expectation in following table.

Demand (Number of T-shirts)	**Probability**
2000	15%
5000	50%
8000	35%

a) If they can sell each T-shirt for $5, construct a decision tree to help them make their decision.
b) What is the best option if the demand is 2,000 T-shirts?
c) What is the best option if the demand is 5,000 T-shirts?
d) What is the best option if the demand is 8,000 T-shirts?
e) Which option is the best for them? What is the expected profit if that decision is made?

11. A software company released a beta version of a software package. It expects a large number of requests from the users for fixing potential bugs in the software. These include crashing, lock up, and incompatibility errors. The company has established a help desk to handle telephone requests. The company trained two groups of software specialists to support the software. Group 1 has just been hired and trained; meanwhile specialists in Group 2 are senior technicians very capable of solving the problems. The senior specialists solve the problems with 100 percent certainty, but their salaries are much higher than other specialists. The payment system of the company for the specialists is problem based and the company pays them based on the number of the problems that they attempt to solve. Group 1 salaries are $20 per problem and Group 2 salaries are $35 per problem. The software company always has a dilemma as to which specialist to assign in order to minimize the cost of the support. If they assign a problem to a Group 1 specialist and he is not able to solve the problem, they reassign it to a senior specialist. In this case pays both specialists are paid. This costs the company $55 per problem. To address this issue, they developed an automatic system to predict the chance of solving a problem by group 1 based on previous cases.

    a) A crashing problem was just received, and the prediction software forecasts a 70 percent chance of success for a Group 1 specialist. Draw a decision tree for this problem.
    b) Based on the decision tree, what kind of specialist should be assigned to the problem first?
    c) Another problem, compatibility error, was received and the prediction software forecasts a 50 percent chance of success for a Group 1 specialist. Draw a decision tree for this decision.
    d) Based on the decision tree, what kind of specialist should be assigned to the problem?
    e) Let p represent the probability that the Group 1 specialist will be able to solve the problem. Write an equation to calculate the expected value of the cost for each decision as a function of p.
    f) Graph the equations of the expected values to determine their intersection point.
    g) They want to know what should be the cutoff value of the probability to assign directly to an expert instead of assigning the task to a group-one specialist. Use both a graphical representation of part e) and an algebraic representation of part f) to find that probability.
    h) In the previous question, what was the role of the two salaries in determining the beak even value of p? Assume that the salary of group one specialists is x and the salary of group two workers is y. Write an equation to calculate the expected value of the cost for each decision as a function of p, x and y.
    i) Find the value of p as a function of x and y that leads to the same expected value of the cost regardless of whether the problem is first assigned to a Group 1 or Group 2 specialist.

12. (Continuing the previous problem) After finishing the first phase, management figured out that they need a Group 2 category of specialists that are more knowledgeable than Group 1 but not necessarily experts. They are to be paid at $28 because of their higher success rates than Group 1.

    a) A problem was just received and the prediction software forecasts 70 percent chance of success for a group-one specialist and 85 percent chance of success for a group-two specialist. Experts can solve the problem for sure. Draw a decision tree for this problem. (Assume that if a group-two specialist fails to fix the problem, the company won't assign it to group one)
    b) Based on the decision tree, what kind of specialist should be assigned to the problem first?
    c) What if the probability of success for Group 1 would be 60% and still 85% for Group 2?

13. An automotive part has to go through two different processes by metal lathes to be shaped properly. Each process has a cost associated with the type of lathe. Each step in processing has a risk of ruining the part and turning it into scrap. For example, when Lathe 1 is processes a part, the cost is $100 and the risk of being scrapped is 10%. Each part that successfully processed by both lathes is sold for $450. The net profit is equal to the number of parts sold minus the cost of processing all parts. The

cost of processing includes both finished and scrapped parts. The following table shows the cost and risk of each lathe.

	Cost	Risk of scrap
Lathe 1	$100	10%
Lathe 2	$150	20%

There is no recycling value if the part is ruined; scrapped parts are worthless.

a) What is the probability that a part will end up being scrapped? Does it make a difference as to the order of the processes?
b) The processes can be done in either order. Draw a decision tree to determine the optimal sequence of processes which maximizes the expected value of the net profit per part.
c) Which process should be done first?
d) If the cost of processing by Lathe 2 changed, for what cost would the optimal strategy change? What is the percentage change?

14. Continuing the previous problem, suppose there are three processes that must be done by three different lathes. Parts that are made with these 3 processes are sold for $800 each. Each lathe has an associated cost and risk of ruin as follows:

	Cost	Risk
Lathe 1	$100	10%
Lathe 2	$150	20%
Lathe 3	$200	25%

a) How many different sequences need to be considered?
b) What is the probability that a part is ruined?
c) Draw a decision tree to determine the optimal sequence of processes which maximizes the expected value of the net profit per part.
d) In which order should the processes be done?
e) Explain how you can use a pair-wise comparison among processes to find the optimal sequence?
f) If there were four processes, how many pair-wise comparisons would need to be made to find the optimal sequence?

Chapter 10 Summary

What have we learned?

We have learned that decision-making often involves uncertainty. It may be easy to choose between two events such as taking a job that pays $8 per hour or a job that pays $10 per hour. However it becomes more difficult to choose if the benefits are less certain. What if there was a chance the higher paying job involved a lower base pay but you could earn tips so your pay was $6 per hour with tips ranging from $0 to $4 per hour. Your decision would need to be based on how likely those amounts were. In order to choose between multiple options, we need to be able to identify not only the benefit of different possibilities but also the likelihood of the different possibilities occurring. A decision tree can help us to combine those together to find the expected values of different options. The option we should choose is the one that has the highest expected value. It is interesting to note that often, the expected value is not actually a possible value.

Using probability trees is a multiple step process:

1. Create a decision tree including decision nodes, chance nodes and end nodes
2. Identify the potential cost or benefit for each branch off a node
3. Identify the probability for each branch off a chance node
4. Find expected value for each branch
5. Combine the expected values for each branch of the decision node
6. Identify the decision node branch with the best expected value

Terms

Probability Tree	A diagram used to show all the possible outcomes for a combination of events
Fundamental Principle of Counting	If there are m possible outcomes for one event and n possible outcomes for a second event then there are $m \times n$ possible ways in which both events can occur
Compound Event	An event that is the result of more than one outcome
Multiplication Rule	If A and B are independent events then the probability of A and B both happening equals the probability of A times the probability of B
Independent	Events A and B are independent if the outcomes for events A and B do not effect each other
Random Variable	A variable whose value depends on random events
Expected Value	The weighted average found by multiplying the possible results of an random variable by the probability of the random variable having that value
Chance Event	An event whose likelihood must be predicted using probability and is outside anyone's control
Decision Tree	A diagram similar to a probability tree to model a situation representing possible scenarios depending on decisions in addition to random events
Node	The boxes (representing decisions), circles (representing random chance), and triangles (representing the end of a path) on the decision tree
Arcs	The connections between nodes on a decision tree showing possible scenarios
Take Rate	The proportion of customers that select a particular option
Risk Averse	A desire to avoid risks even when the potential benefit of an action is large

Chapter 10 (Decision Trees) Objectives

You should be able to:

- Create a decision tree including decision nodes, chance nodes and end nodes
- Identify the potential cost or benefit for each branch off a node
- Identify the probability for each branch off a chance node
- Use the fact that probabilities must add up to 1
- Find value for each branch of the decision tree
- Find the expected value for each option

Chapter 10 Study Guide

1. What is a decision tree and for what is it used?

2. What are nodes on a decision tree? What shape is used for each type of node?

3. How do you find the end node value for a branch of a decision tree? Where do you put that value on the decision tree?

4. Once you find the value for each branch of a decision tree, how do you find the expected value for each option of the decision nodes?

5. What must be true about the sum of the probabilities for each branch after a decision node?

6. What does risk averse mean and how does it affect the results of decision tree analysis?

7. Give an example of risk aversion.

